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Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

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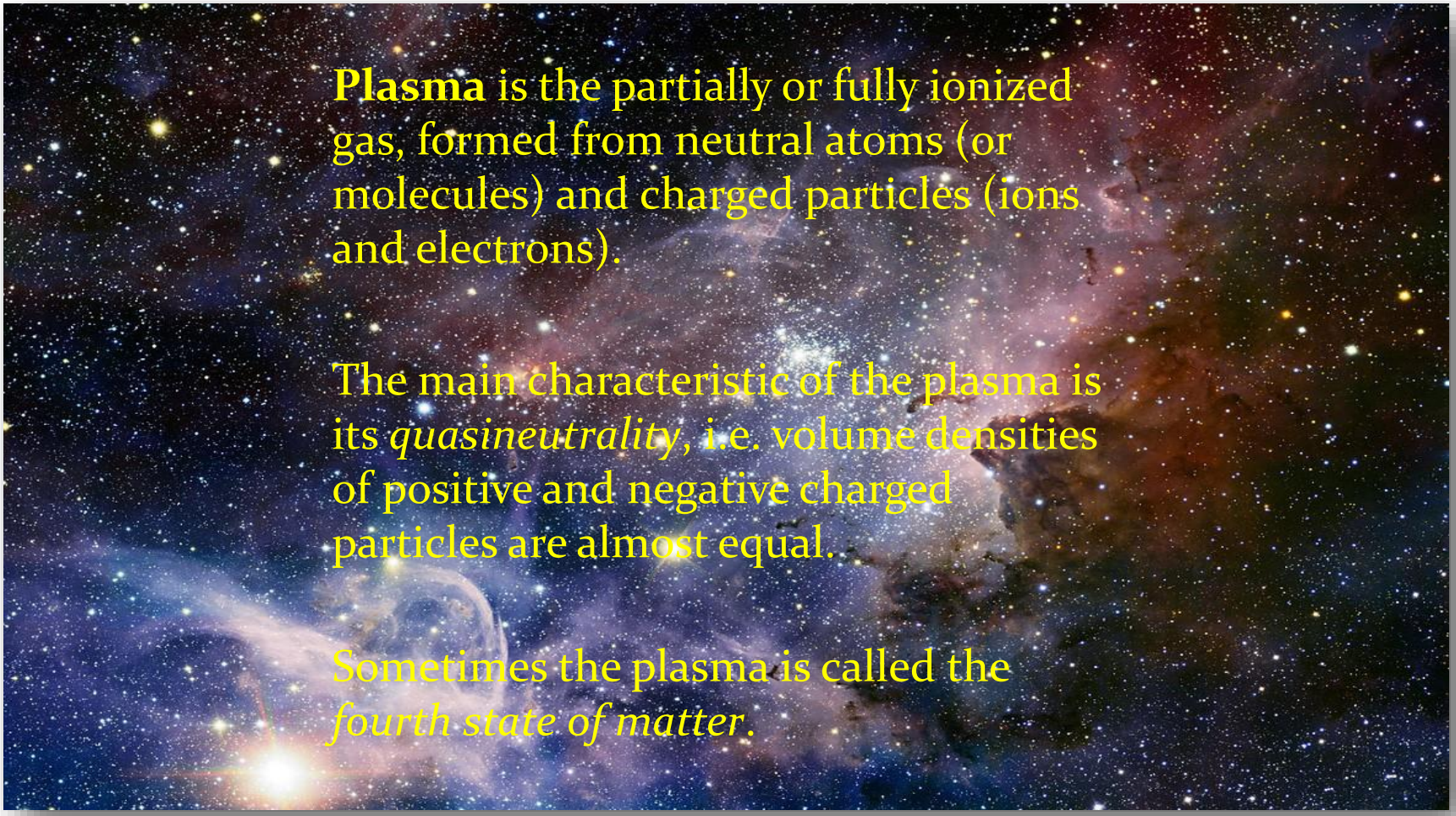


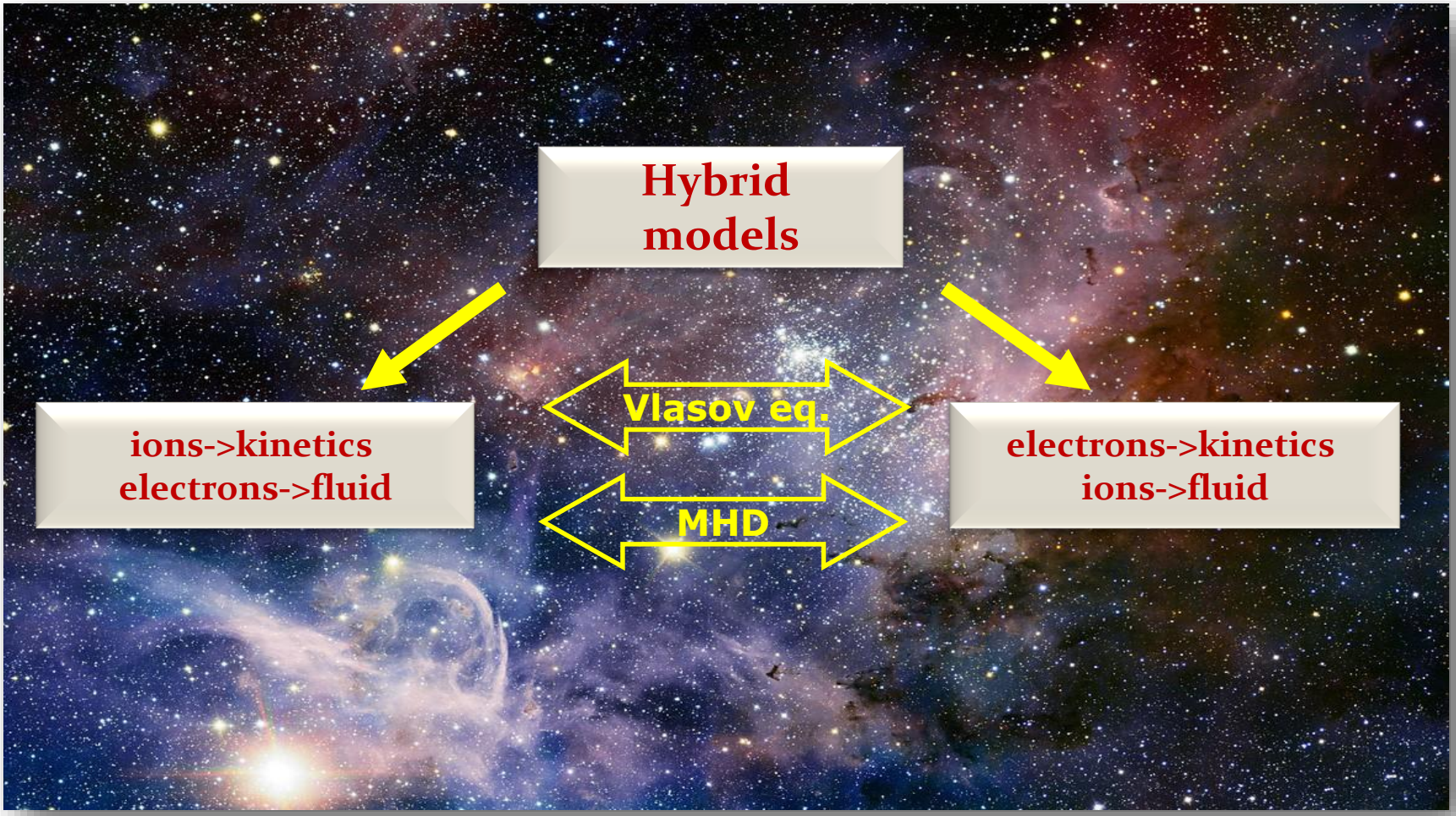
Plasma is the partially or fully ionized gas, formed from neutral atoms (or molecules) and charged particles (ions and electrons).

The main characteristic of the plasma is its *quasineutrality*, i.e. volume densities of positive and negative charged particles are almost equal.

Sometimes the plasma is called the *fourth state of matter*.

Introduction. Definitions

0.2 The term "Hybrid numerical model"

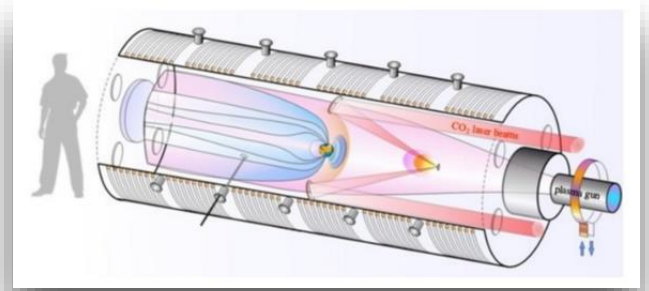


Introduction. Definitions

0.3.2 Implementation of the hybrid numerical models

ions	→	**kinetics**
electrons	→	**fluid**

- Laboratory experiments:



- Supernova

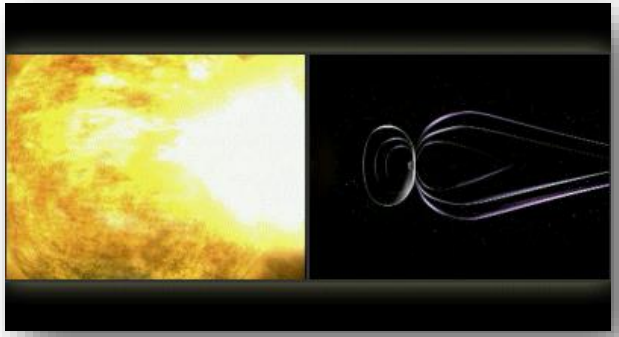
- Cosmic rays

Introduction. Definitions

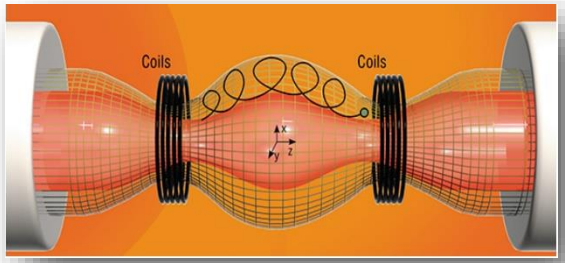
0.3.1 Implementation of the hybrid numerical models

electrons	→	**kinetics**
ions	→	**fluid**

- Northern lights
- Magnetic substorms



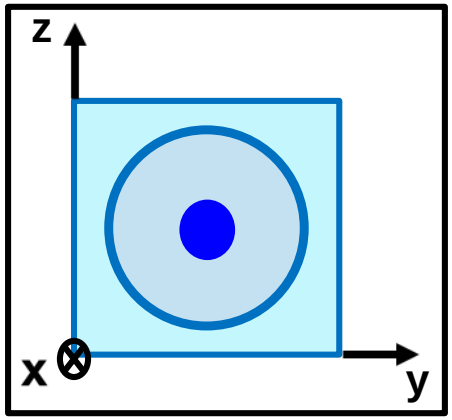
- Laboratory experiments:

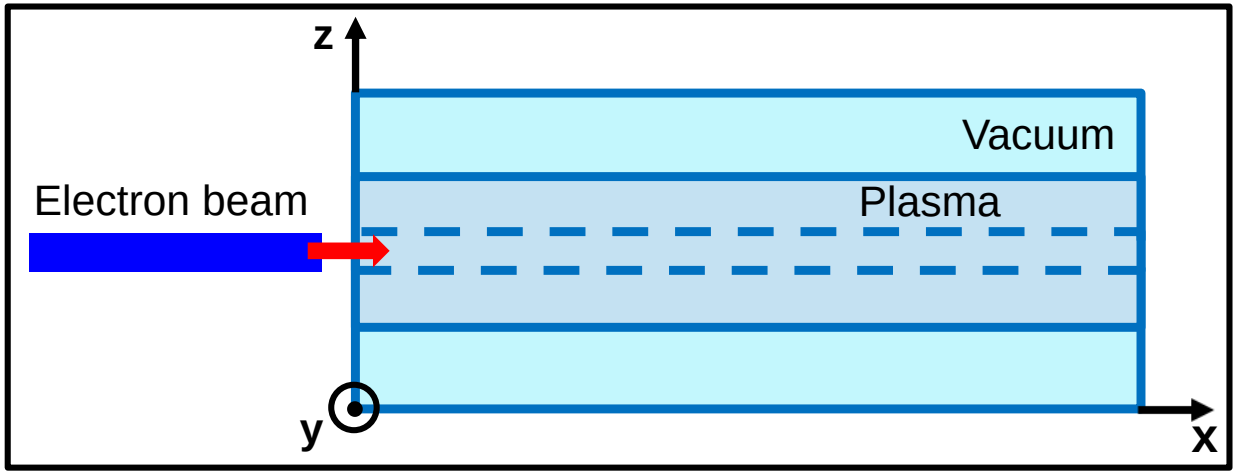


Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

1.1 Geometry and assumptions

- 3D problem;
- Cartesian coordinates;
- plasma consists of the hydrogen ions and electrons;
- uniform magnetic field.





Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

1.2.1 Equations

Equation for electrons (the Vlasov kinetic equation) is

$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{r}} + q\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right)\frac{\partial f}{\partial \vec{\mathrm{p}}} = 0.$$

Here f is an electron distribution function, $\vec{p}$ is a relativistic momentum of the beam particles, q is a charge, m_e is an electron mass, c is the speed of light, $\vec{B} = (B_x, B_y, B_z)$ is a magnetic field intensity, $\vec{E} = (E_x, E_y, E_z)$ is an electric field intensity and $\vec{r} = (x, y, z)$. The characteristics of the Vlasov equation are

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{v}, \qquad \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = -\mathrm{e}\left(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}\right).$$

The electron density and average velocity are

$$n_e(\vec{r}, t) = -\mathrm{e}\int f_e(\vec{r}, \vec{v}, t)\,\mathrm{d}\vec{v},$$

$$\vec{V}_e(\vec{r}, t) = \frac{1}{n_e(\vec{r}, t)}\int \vec{v} f_e(\vec{r}, \vec{v}, t)\,\mathrm{d}\vec{v}.$$

Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

1.2.2 Equations

Equation for ions

$$m_i n_i \frac{\mathrm{d}\vec{V}_i}{\mathrm{d}t} = -\nabla p_i + \mathrm{e} n_i \left(\vec{E} + \frac{1}{c} \vec{V}_i \times \vec{B} \right).$$

Here V_i is an ion velocity, m_i is an ion mass, n_i is an ion density, p_i is an ion pressure. The continuity equation to find an ion density

$$\frac{\mathrm{d}n_i}{\mathrm{d}t} + n_i \nabla \cdot V_i = 0.$$

The electric and magnetic field intensities are computed from the Maxwell equations

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \qquad \left(\vec{j} = \mathrm{e}\left(n_i \vec{V}_i - n_e \vec{V}_e\right)\right).$$

The heat conduction equation to find an ion temperature is

$$n_i \left(\frac{\partial T_i}{\partial t} + \left(\vec{V}_i \cdot \nabla\right) T_i \right) + (\gamma - 1) p_i \nabla \cdot \vec{V}_i = (\gamma - 1)\, \nabla \cdot \vec{q}_i.$$

Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

1.2.3 Equations

$$n_i\left(\frac{\partial T_i}{\partial t}+\left(\vec{V}_i\cdot\nabla\right)T_i\right)+(\gamma-1)p_i\nabla\cdot\vec{V}_i=(\gamma-1)\nabla\cdot\vec{q}_i.$$

Here $\vec{q}_i=-k_1\nabla T_i$, where k_1 is a heat conduction coefficient, $\vec{V}_i=\left(V_x,V_y,V_z\right)$ is an average ion velocity.

1.3 Normalization

$$n_0,\qquad c,\qquad B_0=c\sqrt{4\pi n_0 m_e},\qquad T_0,$$

$$L=\frac{c}{\omega_{pe}}=\frac{c\sqrt{m_e}}{\mathrm{e}\sqrt{4\pi n_0}},\qquad t_0=\frac{1}{\omega_{pe}}=\frac{\sqrt{m_e}}{\sqrt{4\pi n_0\mathrm{e}^2}}$$

Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

1.4 Initial and boundary conditions

Initial data of the background plasma:

$$t = 0: \quad n = n_0 = const,\ B_x = B_0 = const,\ B_y = B_z = 0,\ E_x = E_y = E_z = 0,$$

$$V_{ex} = V_{ey} = V_{ez} = 0,\ V_x = V_y = V_z = 0.$$

Boundary conditions are

- Electromagnetic fields:

  by y, z there are periodic boundary conditions;

  at $x = 0,\ x = x_{max}$ the radiation leaves the domain;

  $$\frac{\partial B_x}{\partial x} = 0,\ B_y = B_z = 0,\ E_x = 0,\ \frac{\partial E_y}{\partial x} = \frac{\partial E_z}{\partial x} = 0.$$

- Electron beam ($n_{beam} \ll n_0$):

  at $x = 0$: $V_{ex} = const,\ V_{ey} = V_{ez} = 0,$

  at $x = x_{max}$ the beam particles leave the computational domain.

- Plasma:

  at $x = 0,\ x = x_{max}$ there is a reflection of particles.

1.5.1 Algorithm

$$\frac{\vec{B}^m - \vec{B}^{m-1/2}}{\tau/2} = -\nabla \times \vec{E}^m$$

$$\frac{\vec{V}_i^{m+1/2} - \vec{V}_i^{m-1/2}}{\tau} + \left(\vec{V}_i^{m-1/2} \cdot \nabla\right) \vec{V}_i^{m-1/2} = \frac{1}{2n_i} \nabla (n_i^m T_i^m) + \beta \left(\vec{E}^m + \vec{V}_i^{m-1/2} \times \vec{B}^m\right)$$

$$\frac{n_i^{m+1} - n_i^m}{\tau} + n_i^m \nabla \cdot \vec{V}_i^{m+1/2} + \vec{V}_i^{m+1/2} \cdot \nabla n_i^m = 0$$

$$\frac{\vec{p}^{m+1/2} - \vec{p}^{m-1/2}}{\tau} = -\vec{E}^m - \vec{v}^m \times \vec{B}^m$$

$$\frac{\vec{r}^{m+1} - \vec{r}^m}{\tau} = \vec{v}^{m+1/2}$$

$$\vec{n}_e^{m+1}(\vec{r}) = -q \sum_j R\left(\vec{r}, \vec{r}_j^{m+1}\right)$$

$$(n_e V_e)^{m+1/2}(\vec{r}) = \sum_j \vec{v}_j^{m+1/2} R\left(\vec{r}, \vec{r}_j^{m+1}\right)$$

$$\frac{\vec{B}^{m+1/2} - \vec{B}^m}{\tau/2} = -\nabla \times \vec{E}^m$$

$$\frac{\vec{E}^{m+1} - \vec{E}^m}{\tau} = \nabla \times \vec{B}^{m+1/2} - n_i^{m+1} \vec{V}_i^{m+1/2} + \left(n_e \vec{V}_e\right)^{m+1/2}$$

$$n_i^{m+1} \left(\frac{T^{m+1} - T^m}{\tau} + \left(\vec{V}_i^{m+1/2} \cdot \nabla\right) T^m\right) + (\gamma - 1) n_i^{m+1} T_i^m \nabla \cdot \vec{V}_i^{m+1/2} = (\gamma - 1) \nabla \cdot \vec{q}_i^m$$

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1.5.2 Algorithm

A uniform grid with steps h_x, h_y, h_z by the axis x, y, z, accordingly, is introduced. Grid functions are defined in the following nodes

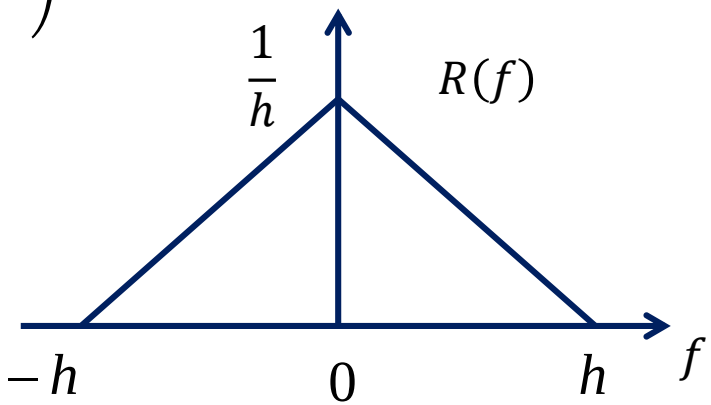
$$B_x \to \left(i - \frac{1}{2}, j, k\right) \qquad V_x, V_{ex}, E_x \to \left(i, j - \frac{1}{2}, k - \frac{1}{2}\right) \qquad n, T \to \left(i - \frac{1}{2}, j - \frac{1}{2}, k - \frac{1}{2}\right)$$

$$B_y \to \left(i, j - \frac{1}{2}, k\right) \qquad V_y, V_{ey}, E_y \to \left(i - \frac{1}{2}, j, k - \frac{1}{2}\right)$$

$$B_z \to \left(i, j, k - \frac{1}{2}\right) \qquad V_z, V_{ez}, E_z \to \left(i - \frac{1}{2}, j - \frac{1}{2}, k\right)$$

Here $R(\vec{r}) = R(x)R(y)R(z)$

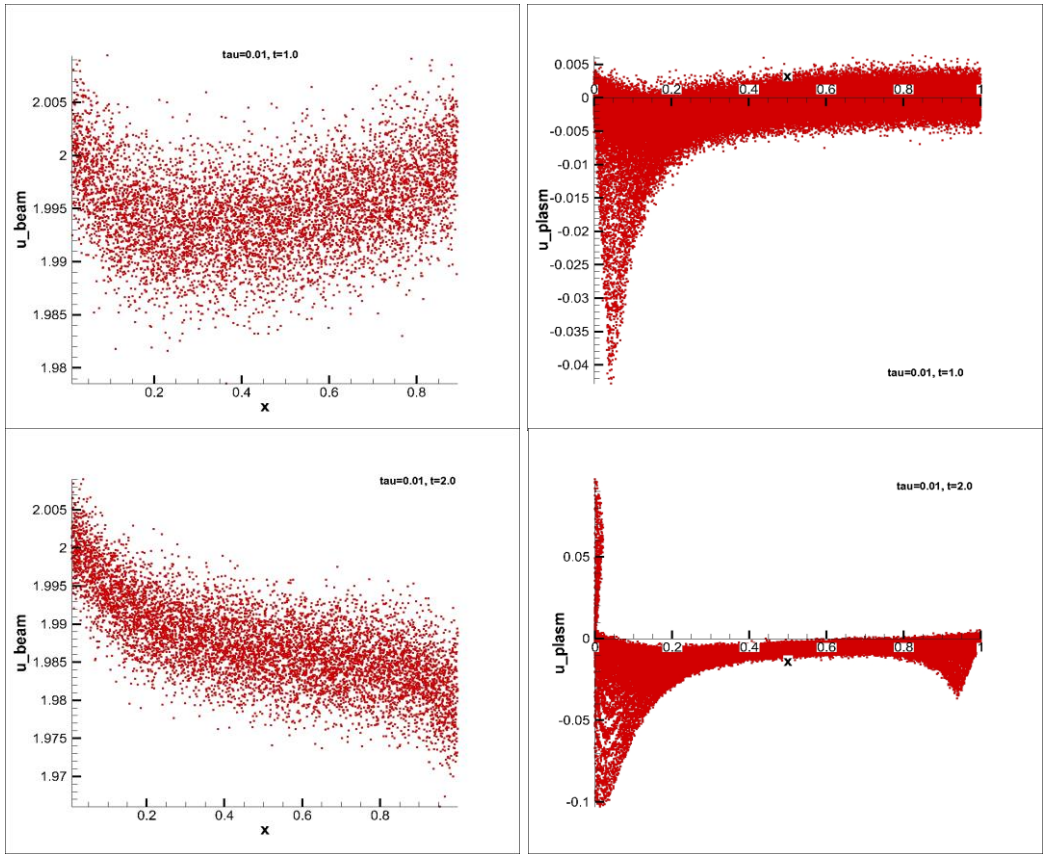
$$R(f) = \begin{cases} 1 - \dfrac{|f|}{h}, & |f| \le h \\ 0, & |f| > h, \end{cases} \qquad f = \{x, y, z\}$$



Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

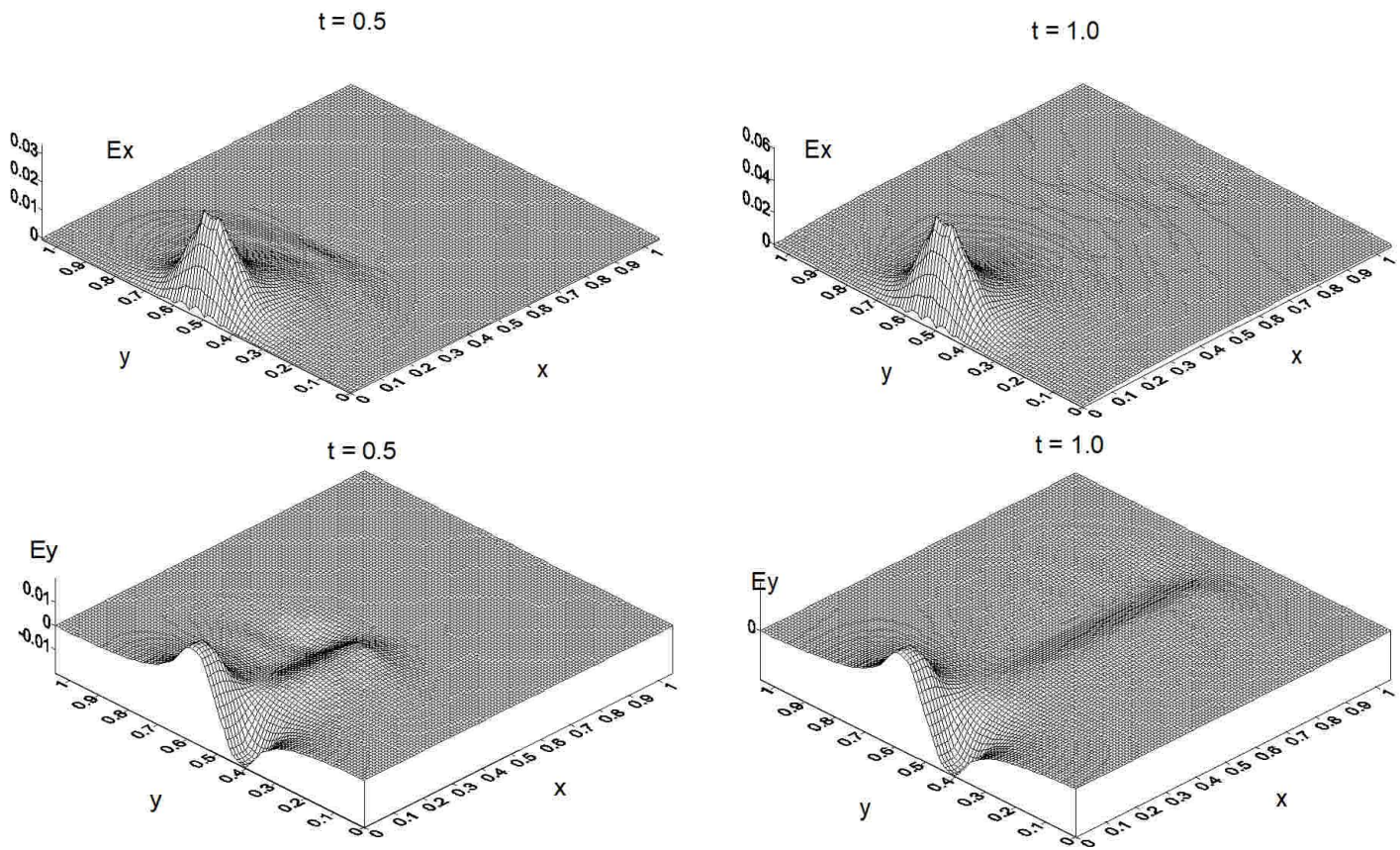
2.1 Results

Phase spaces of the incoming electron beam and the plasma



Kinetic-MHD numerical model of the interaction of an electron beam with the plasma
2.2 Results

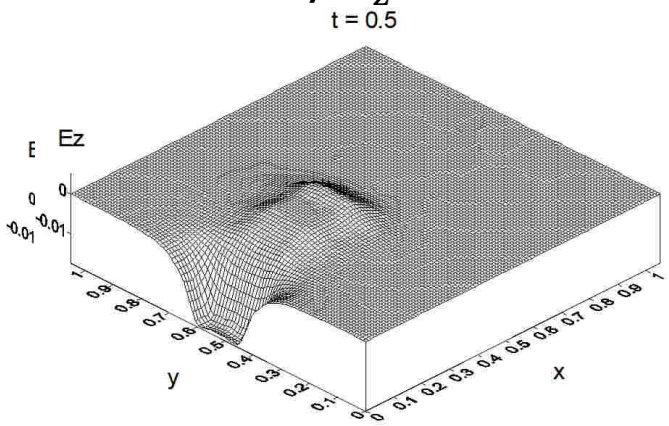
Electric field intensities E_x, E_y:

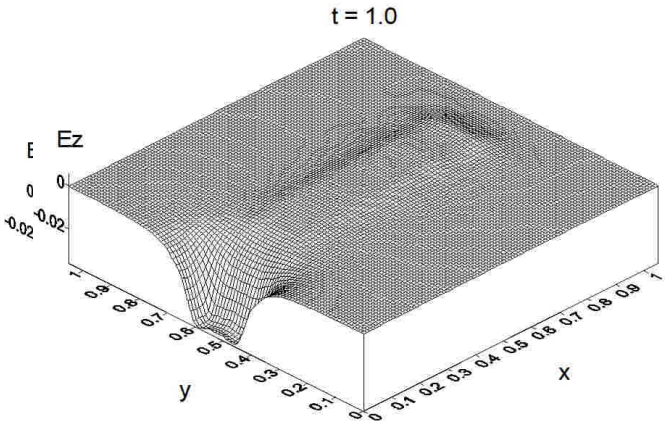


Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

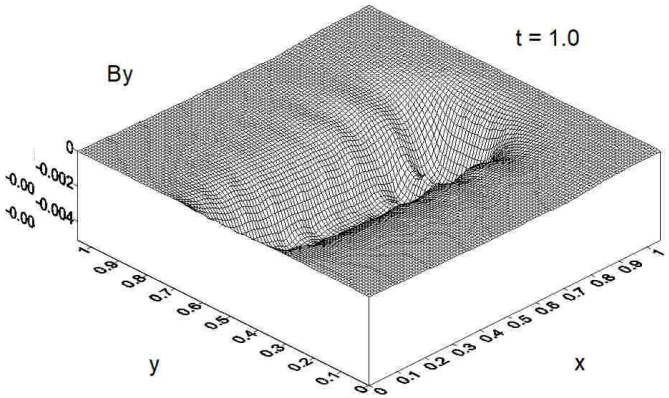
2.3 Results

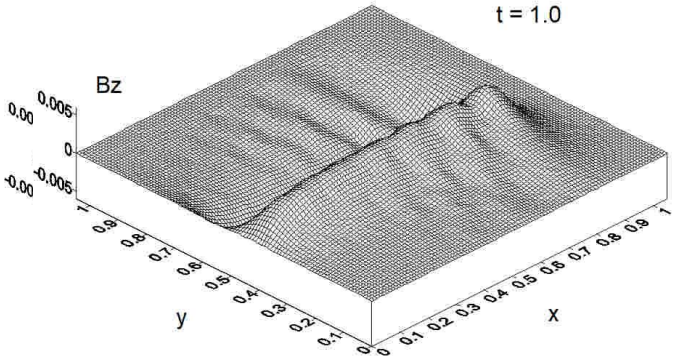
Electric field intensity E_z :





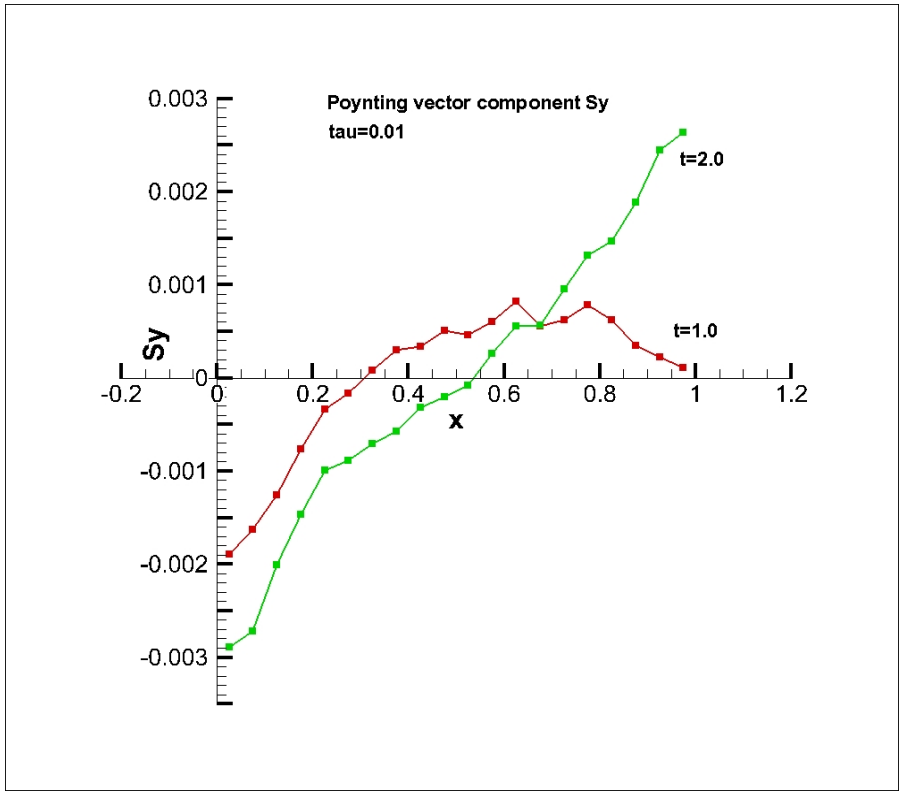
Magnetic field intensities B_y, B_z:





Kinetic-MHD numerical model of the interaction of an electron beam with the plasma
2.4 Results

Component S_y of the Poynting vector $\vec{S} = \frac{c}{8\pi}\vec{E} \times \vec{B}$ in the plane (x, y) at the fixed z



Conclusion

- There has been developed the 3D hybrid numerical model to solve the problem on the electromagnetic wave generation as a result of the interaction of an electron beam and the plasma.

- The aim is to study the influence of the plasma density and the electron beam characteristics to generate the radiation with the given parameters (the terahertz radiation). It is necessary to study the properties of different materials, etc.

- In the coming research the comparison with a fully kinetic model will provide us with the limits of use of the developed model.

Thank you for your attention!