Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

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Introduction. Definitions

0.1 Plasma

Plasma is the partially or fully ionized gas, formed from neutral atoms (or molecules) and charged particles (ions and electrons).

The main characteristic of the plasma is its quasineutrality, i.e. volume densities of positive and negative charged particles are almost equal.

Sometimes the plasma is called the fourth state of matter.
0.2 The term “Hybrid numerical model”

**Introduction. Definitions**

**Hybrid models**

- ions $\rightarrow$ kinetics
- electrons $\rightarrow$ fluid

- Vlasov eq

- MHD

- electrons $\rightarrow$ kinetics
- ions $\rightarrow$ fluid
Introduction. Definitions

0.3.2 Implementation of the hybrid numerical models

- ions ➔ kinetics
- electrons ➔ fluid

➢ Laboratory experiments:

- Supernova
- Cosmic rays
Introduction. Definitions

0.3.1 Implementation of the hybrid numerical models

- Northern lights
- Magnetic substorms

Electrons ➔ Kinetics

Ions ➔ Fluid

Laboratory experiments:
Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

1.1 Geometry and assumptions

- 3D problem;
- Cartesian coordinates;
- plasma consists of the hydrogen ions and electrons;
- uniform magnetic field.

![Diagram of the interaction model](image)
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1.2.1 Equations

Equation for electrons (the Vlasov kinetic equation) is

\[
\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + q \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \frac{\partial f}{\partial p} = 0.
\]

Here \( f \) is an electron distribution function, \( \mathbf{p} \) is a relativistic momentum of the beam particles, \( q \) is a charge, \( m_e \) is an electron mass, \( c \) is the speed of light, \( \mathbf{B} = (B_x, B_y, B_z) \) is a magnetic field intensity, \( \mathbf{E} = (E_x, E_y, E_z) \) is an electric field intensity and \( \mathbf{r} = (x, y, z) \). The characteristics of the Vlasov equation are

\[
\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad \frac{d\mathbf{p}}{dt} = -e \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right).
\]

The electron density and average velocity are

\[
n_e(\mathbf{r}, t) = -e \int f_e(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v},
\]

\[
\mathbf{V}_e(\mathbf{r}, t) = \frac{1}{n_e(\mathbf{r}, t)} \int \mathbf{v} f_e(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v}.
\]
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1.2.2 Equations

Equation for ions

\[ m_i n_i \frac{d\vec{V}_i}{dt} = -\nabla p_i + e n_i \left( \vec{E} + \frac{1}{c} \vec{V}_i \times \vec{B} \right). \]

Here \( V_i \) is an ion velocity, \( m_i \) is an ion mass, \( n_i \) is an ion density, \( p_i \) is an ion pressure. The continuity equation to find an ion density

\[ \frac{dn_i}{dt} + n_i \nabla \cdot V_i = 0. \]

The electric and magnetic field intensities are computed from the Maxwell equations

\[ \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}, \quad \nabla \times \vec{B} = \frac{4\pi}{c} j + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \quad \left( j = e(n_i \vec{V}_i - n_e \vec{V}_e) \right). \]

The heat conduction equation to find an ion temperature is

\[ n_i \left( \frac{\partial T_i}{\partial t} + (\vec{V}_i \cdot \nabla)T_i \right) + (\gamma - 1) p_i \nabla \cdot \vec{V}_i = (\gamma - 1) \nabla \cdot \vec{q}_i. \]
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1.2.3 Equations

\[ n_i \left( \frac{\partial T_i}{\partial t} + (\vec{V}_i \cdot \nabla)T_i \right) + (\gamma - 1)p_i \nabla \cdot \vec{V}_i = (\gamma - 1)\nabla \cdot \vec{q}_i. \]

Here \( \vec{q}_i = -k_1 \nabla T_i \), where \( k_1 \) is a heat conduction coefficient, \( \vec{V}_i = (V_x, V_y, V_z) \) is an average ion velocity.

1.3 Normalization

\[ n_0, \quad c, \quad B_0 = c\sqrt{4\pi n_0 m_e}, \quad T_0, \]

\[ L = \frac{c}{\omega_{pe}} = \frac{c\sqrt{m_e}}{e\sqrt{4\pi n_0}}, \quad t_0 = \frac{1}{\omega_{pe}} = \frac{\sqrt{m_e}}{\sqrt{4\pi n_0 e^2}}. \]
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1.4 Initial and boundary conditions

Initial data of the background plasma:

\[ t = 0: \quad n = n_0 = \text{const}, \quad B_x = B_0 = \text{const}, \quad B_y = B_z = 0, \quad E_x = E_y = E_z = 0, \]
\[ V_{ex} = V_{ey} = V_{ez} = 0, \quad V_x = V_y = V_z = 0. \]

Boundary conditions are:

- Electromagnetic fields:
  - by \( y, z \) there are periodic boundary conditions;
  - at \( x = 0, \quad x = x_{\text{max}} \) the radiation leaves the domain;
  - \[ \frac{\partial B_x}{\partial x} = 0, \quad B_y = B_z = 0, \quad E_x = 0, \quad \frac{\partial E_y}{\partial x} = \frac{\partial E_z}{\partial x} = 0. \]

- Electron beam \( (n_{\text{beam}} \ll n_0) \):
  - at \( x = 0: \quad V_{ex} = \text{const}, \quad V_{ey} = V_{ez} = 0, \)
  - at \( x = x_{\text{max}} \) the beam particles leave the computational domain.

- Plasma:
  - at \( x = 0, \quad x = x_{\text{max}} \) there is a reflection of particles.
1.5.1 Algorithm

\[
\frac{B^m - B^{m-1/2}}{\tau/2} = -\nabla \times \hat{E}^m
\]

\[
\frac{\nabla (n_i^m T_i^m)}{2n_i} + \beta \left( \hat{E}^m + \hat{V}_i^{m-1/2} \times \hat{B}^m \right)
\]

\[
\frac{\bar{V}_i^{m+1/2} - \bar{V}_i^{m-1/2}}{\tau} + \left( \bar{V}_i^{m-1/2} \cdot \nabla \right) \bar{V}_i^{m-1/2} = \frac{1}{2n_i} \nabla (n_i^m T_i^m) + \beta \left( \hat{E}^m + \hat{V}_i^{m-1/2} \times \hat{B}^m \right)
\]

\[
\frac{n_i^{m+1} - n_i^m}{\tau} + n_i^m \nabla \cdot \bar{V}_i^{m+1/2} + \bar{V}_i^{m+1/2} \cdot \nabla n_i^m = 0
\]

\[
\frac{\bar{p}_i^{m+1/2} - \bar{p}_i^{m-1/2}}{\tau} = -\hat{E}^m - \hat{v}_i^m \times \hat{B}^m
\]

\[
\frac{\bar{r}_i^{m+1} - \bar{r}_i^m}{\tau} = \hat{v}_i^{m+1/2}
\]

\[
\bar{n}_e^{m+1}(\bar{r}) = -q \sum_j R(\bar{r}, \bar{r}_j^{m+1})
\]

\[
(n_e V_e)^{m+1/2}(\bar{r}) = \sum_j \hat{v}_j^{m+1/2} R(\bar{r}, \bar{r}_j^{m+1})
\]

\[
\frac{\bar{B}^{m+1/2} - \bar{B}^m}{\tau/2} = -\nabla \times \hat{E}^m
\]

\[
\frac{\bar{E}_i^{m+1} - \bar{E}_i^m}{\tau} = \nabla \times \bar{B}^{m+1/2} - \frac{n_i^{m+1} \bar{V}_i^{m+1/2} + (n_e V_e)^{m+1/2}}{\tau}
\]

\[
n_i^{m+1} \left( \frac{T_i^{m+1} - T_i^m}{\tau} + \left( \bar{V}_i^{m+1/2} \cdot \nabla \right) T_i^m \right) + (\gamma - 1) n_i^{m+1} T_i^m \nabla \cdot \bar{V}_i^{m+1/2} = (\gamma - 1) \nabla \cdot \bar{q}_i^m
\]
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1.5.2 Algorithm

A uniform grid with steps $h_x, h_y, h_z$ by the axis $x, y, z$, accordingly, is introduced. Grid functions are defined in the following nodes

$$B_x \rightarrow \left( i - \frac{1}{2}, j, k \right) \quad V_x, V_{ex}, E_x \rightarrow \left( i, j - \frac{1}{2}, k - \frac{1}{2} \right) \quad n, T \rightarrow \left( i - \frac{1}{2}, j - \frac{1}{2}, k - \frac{1}{2} \right)$$

$$B_y \rightarrow \left( i, j - \frac{1}{2}, k \right) \quad V_y, V_{ey}, E_y \rightarrow \left( i - \frac{1}{2}, j, k - \frac{1}{2} \right)$$

$$B_z \rightarrow \left( i, j, k - \frac{1}{2} \right) \quad V_z, V_{ez}, E_z \rightarrow \left( i - \frac{1}{2}, j - \frac{1}{2}, k \right)$$

Here $R(\vec{r}) = R(x)R(y)R(z)$

$$R(f) = \begin{cases} 1 - \frac{|f|}{h}, & |f| \leq h \\ 0, & |f| > h \end{cases} \quad f = \{x, y, z\}$$
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2.1 Results

Phase spaces of the incoming electron beam and the plasma
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2.2 Results

Electric field intensities $E_x, E_y$:
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2.3 Results

Electric field intensity $E_z$:

Magnetic field intensities $B_y, B_z$:
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2.4 Results

Component $S_y$ of the Poynting vector $\vec{S} = \frac{c}{8\pi} \vec{E} \times \vec{B}$ in the plane $(x, y)$ at the fixed $z$
Conclusion

➢ There has been developed the 3D hybrid numerical model to solve the problem on the electromagnetic wave generation as a result of the interaction of an electron beam and the plasma.

➢ The aim is to study the influence of the plasma density and the electron beam characteristics to generate the radiation with the given parameters (the terahertz radiation). It is necessary to study the properties of different materials, etc.

➢ In the coming research the comparison with a fully kinetic model will provide us with the limits of use of the developed model.
Thank you for your attention!