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Information-Based Model Reduction for Nonlinear Electro-Quasistatics Field Problems

Fotios Kasolis, Markus Clemens

Chair of Electromagnetic Theory

School of Electrical, Information, and Media Engineering

University of Wuppertal

Grant sponsor: German Research Foundation (No. CL143/10-2)

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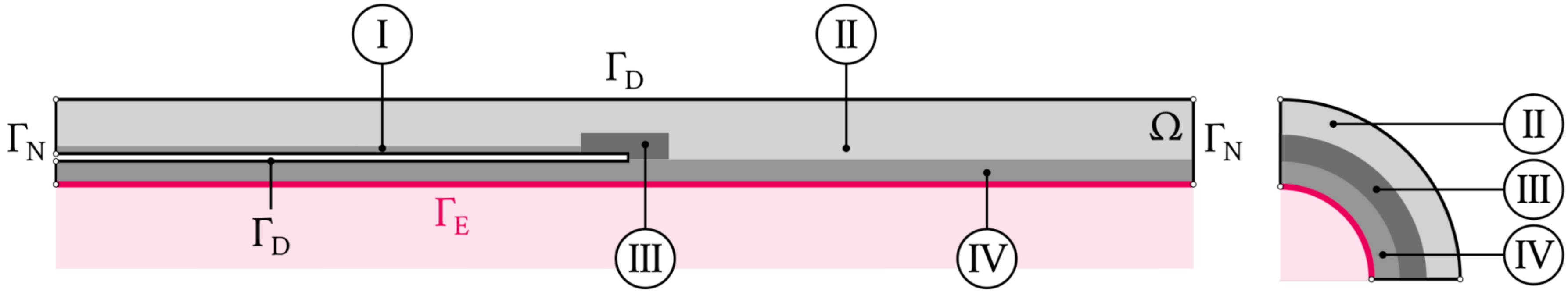
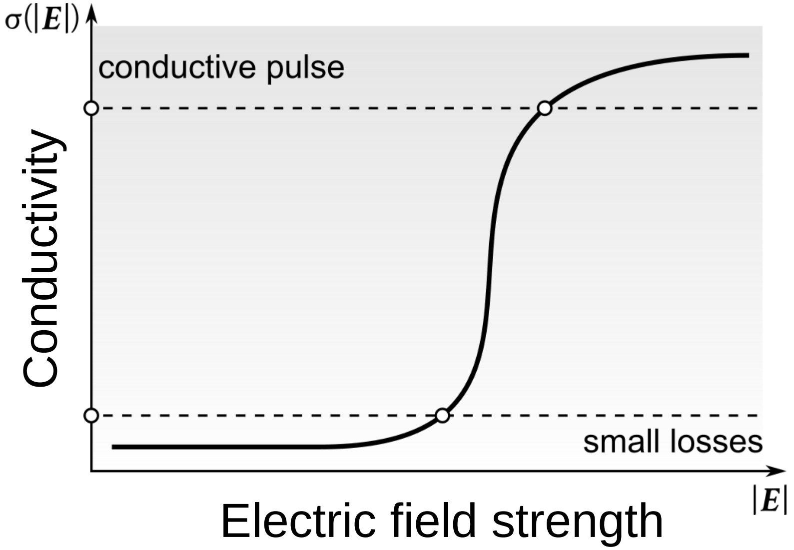
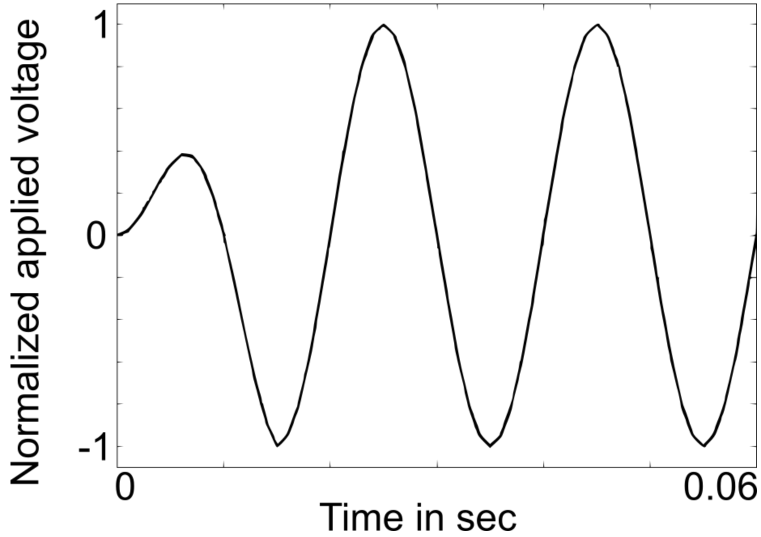
Problem statement

$$\nabla \cdot [\sigma(|\nabla\phi|)\nabla\phi] + \nabla \cdot \left[\epsilon \nabla \frac{\partial \phi}{\partial t}\right] = 0 \text{ in } \Omega$$

$$\gamma_{\Gamma_D}\phi = 0, \qquad \gamma_{\Gamma_E}\phi = g(t), \qquad \partial_n \phi = 0 \text{ on } \Gamma_N$$







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Problem statement

- **Field grading material exhibit switch-like behavior**
  - **Up to moderate field strength values: non-conductive**
  - **High field strength values: conductive**

Goal: accurate and efficient model reduction for transient EQS problems with localized strong nonlinearities

- **High-fidelity discretization (FEM and implicit time)**

$$\boldsymbol{F}_{k+1}(\boldsymbol{\phi}_{k+1}) = \mathbf{0}, \qquad \boldsymbol{\phi}_{k+1} \in \mathbb{R}^{m}, \qquad \forall k \in \mathbb{N}$$

- **Complexity reduction through reduced basis MR**

$$\widetilde{\boldsymbol{F}}_{k+1}(\widetilde{\boldsymbol{\phi}}_{k+1}) = \mathbf{0}, \qquad \widetilde{\boldsymbol{\phi}}_{k+1} \in \mathbb{R}^{\widetilde{m}}, \qquad \forall k \in \mathbb{N}$$

with $\widetilde{m} \ll m$.

- Nonlinear resistive electric field grading part 2, Materials and applications; L. Donzel, F. Greuter, and T. Christen, ABB Corporate Research, Baden-Dättwil; Switzerland, IEEE Electrical Insulation Magazine

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Model order reduction toolbox

- **Common model reduction strategies are inappropriate**
  - **due to strongly nonlinear conductivity**
- **Proper orthogonal decomposition (POD)**
  - **Snapshots method:** obtain reduced basis using SVD on a small set of high-fidelity solutions (snapshots)
  - **Jacobian evaluation:** depends on the size of the high-fidelity problem
- **Discrete empirical interpolation method (DEIM)**
  - **POD + Greedy selection of interpolation points**
- **Choose reduced number of nodes in a greedy approach to reconstruct the Jacobian**
  - **Greedy selection suffers from the strongly nonlinear conductivity**

- S. Chaturantabut and D. C. Sorensen, “Nonlinear model reduction via discrete empirical interpolation” , SIAM J. Sci. Comput., vol. 32, no. 5, pp. 2737–2764, 2010
- D. Schmidthausler and M. Clemens, “Low-order electroquasistatic field simulations based on proper orthogonal decomposition”, IEEE Trans. Magn, vol. 48, no. 2, pp. 567–570, 2012.

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Model order reduction toolbox

- **Obtain snapshots** $\boldsymbol{\Phi} = [\boldsymbol{\phi}_1 \mid \boldsymbol{\phi}_2 \mid \dots \mid \boldsymbol{\phi}_n] \approx \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^\top$
- **Projection claim** $\boldsymbol{\phi} \approx \boldsymbol{\phi}_r = \mathbf{U}\boldsymbol{\psi},\ \mathbf{U} \in \mathbb{R}^{m\times r},\ \boldsymbol{\psi} \in \mathbb{R}^r,\ r \ll n$
- **Impose** r **interpolation constraints** $\mathbf{U}_r\boldsymbol{\psi} = \boldsymbol{\phi}$
- **Obtain reduced solution vector** $\boldsymbol{\phi}_r = \mathbf{U}\mathbf{U}_r^{-1}\boldsymbol{\phi}$

How to select the locations to impose the constraints?

Greedy selection of interpolation nodes

- $\mathbb{I} \leftarrow \{\mathrm{argmax}(|\boldsymbol{u}_1|)\}$
- $\forall k \in \{2, \dots, r\}$
  - Compute current residual $\boldsymbol{e} \leftarrow \boldsymbol{u}_k - \mathbf{U}[\mathbf{U}(\mathbb{I}, :)]^{-1}\boldsymbol{u}_k(\mathbb{I})$
  - Select worst approximation node $\mathbb{I} \leftarrow \{\mathbb{I}, \mathrm{argmax}(|\boldsymbol{e}|)\}$

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Information and entropy

- **High entropy**
  - **Unavailable energy, disorganization, equally probable events, uniform distribution, unpredictability, much information for state specification**
- **Low entropy**
  - **Available energy, high organization degree, highly uneven distribution, high reliability, little information for state specification**
- **Definition: Shannon's (or information) entropy**
  - **Given signal $\boldsymbol{\phi} = [\phi_1, \dots, \phi_n]^\top$ obtain probabilities $\boldsymbol{p} = [p_1, \dots, p_n]^\top$**

$$H(\boldsymbol{\phi}) = -\sum_{k=1}^{n} p_k \log p_k \in \mathbb{R}$$

  - **A statistic that characterizes an ensemble of probabilities and does not depend on the actual values of the state function**
  - **$H \to 0$, no information supply in time: equilibrium and periodic**
  - **$H \in (0, \infty)$, steady supply of information: aperiodic but deterministic**

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Information and entropy

Grid	Number of states N	Probability $1/N$	Entropy H
	1	1	0
	2	0.5	1
	4	0.25	2
	8	0.125	3
	16	0.062	4

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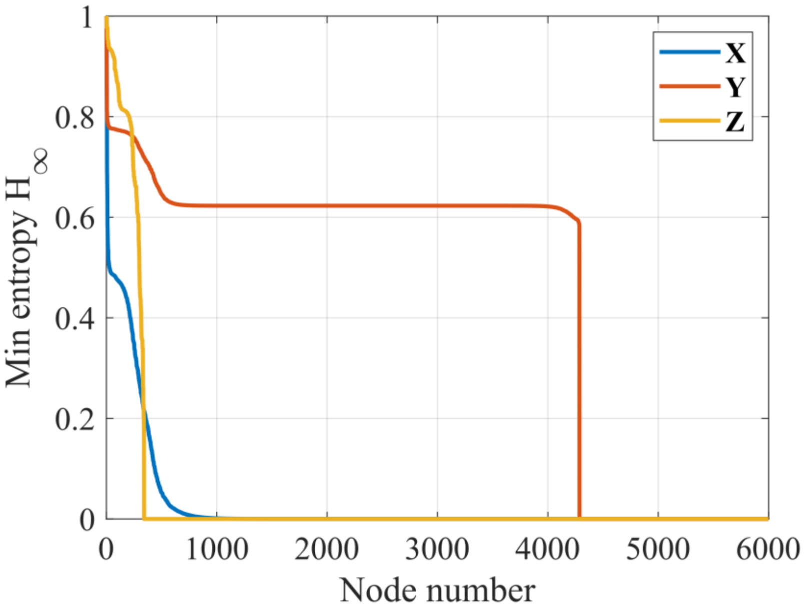
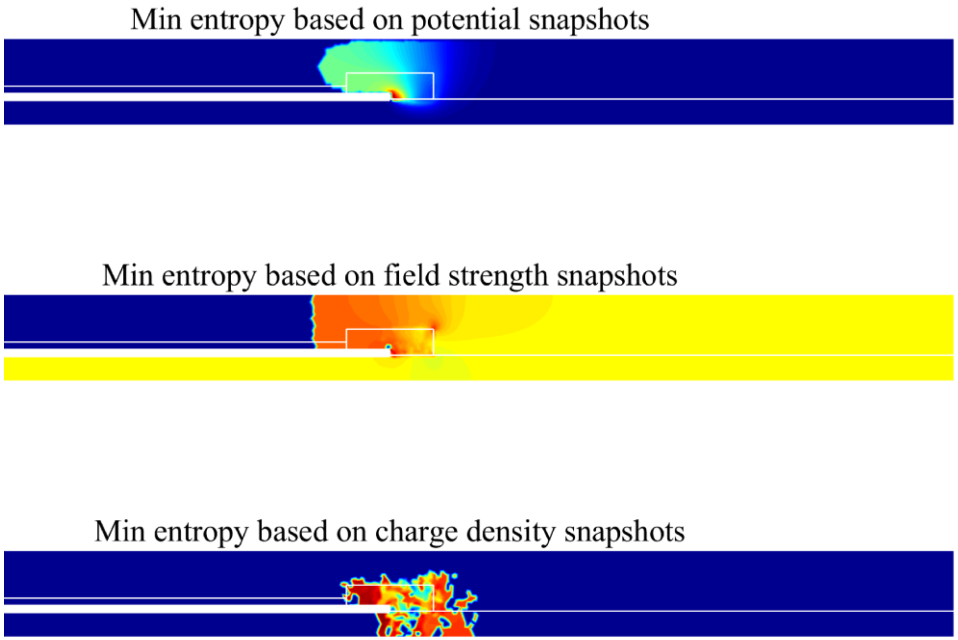
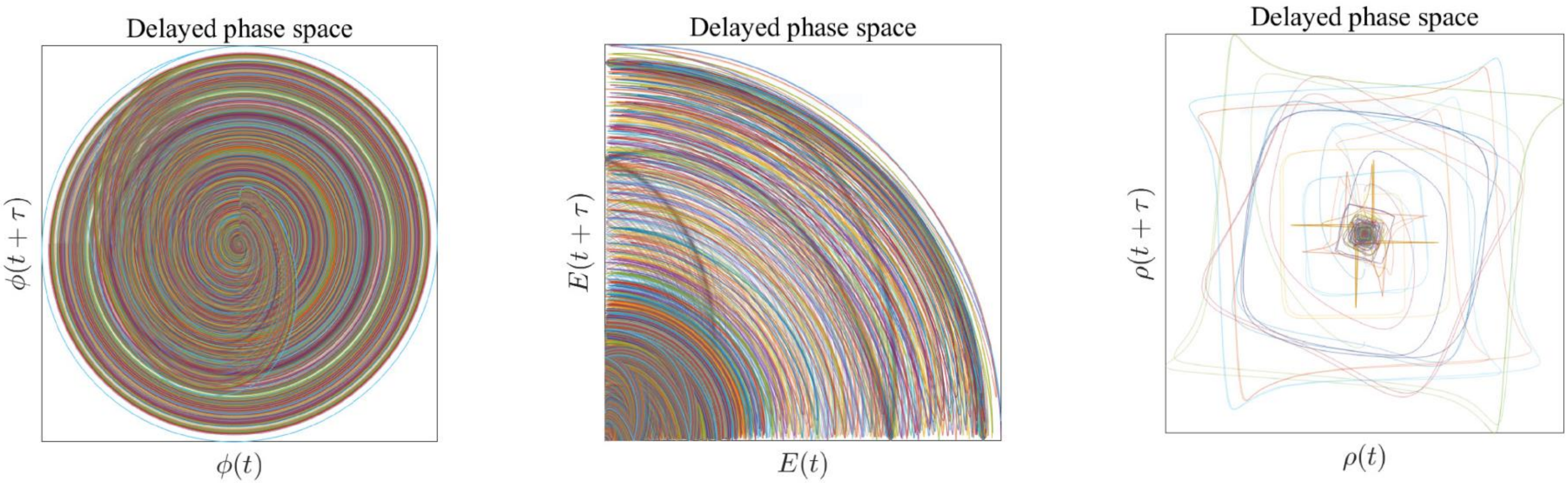
Information and entropy







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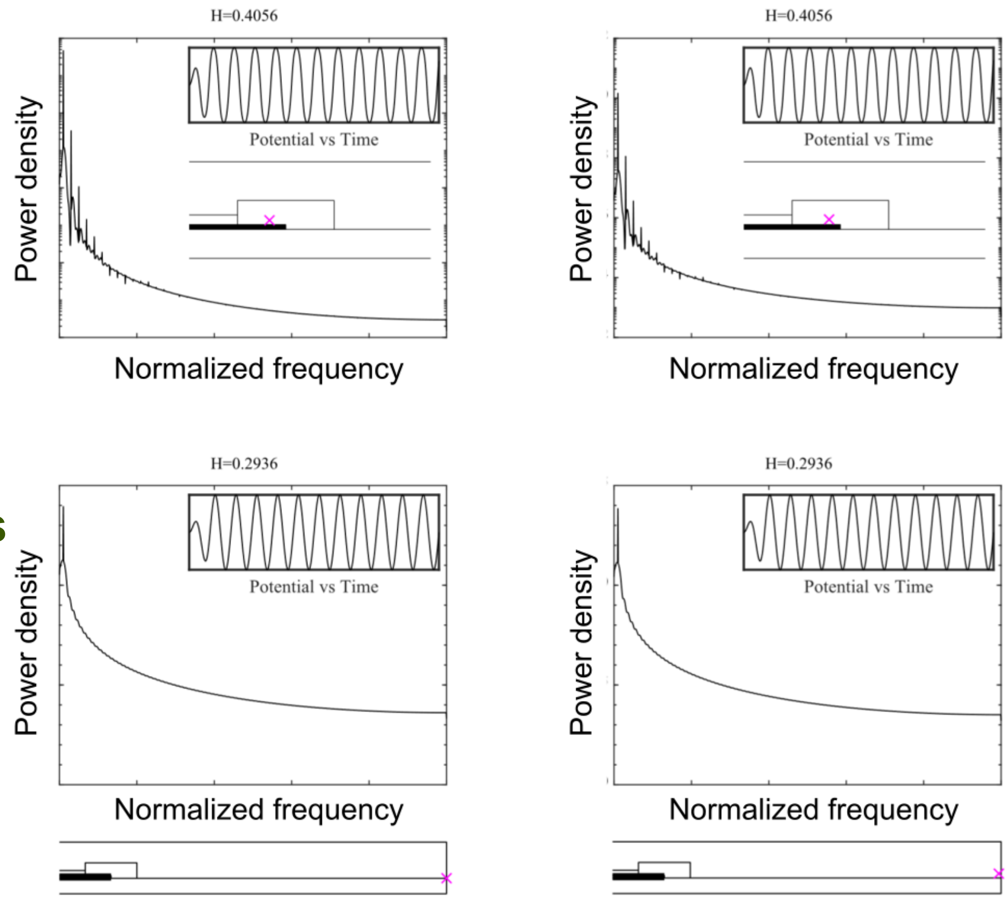
Information and entropy

- Many frequency spectrum
- Weakly periodic time series
- Only short-term computable
- Unstable phase space manifold
- Non-accurate MR with DEIM

- Isolated frequency components
- Periodic time series
- Long-term computable
- Stable phase space manifold
- Accurate MR with DEIM



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Maximal information refinement

- **Kullback-Leibler divergence**
  - **Given signals x, y obtain probabilities $p^{(x)}, p^{(y)}$**

$$D_{\mathrm{KL}}(\boldsymbol{x} \| \boldsymbol{y}) = \sum_{k=1}^{n} p_k^{(\boldsymbol{x})} \log \frac{p_k^{(\boldsymbol{x})}}{p_k^{(\boldsymbol{y})}} \in [0, \infty]$$

  - **Not a metric, but a suitable measure of statistical discrepancy**
  - **The Kullback-Leibler divergence can be viewed as a measure of information loss induced by the use of y in place of x**

- **Remark**
  - **Find nodes with state signals with high entropy H values**
    to capture regions with strongly nonlinear material
  - **Compare these nodes using D_{KL}**
    assuming that they can not be well-represented by the high entropy signals
  - **Impose constraints**

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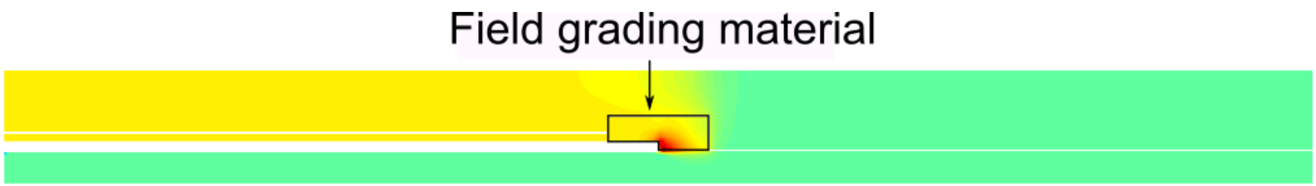
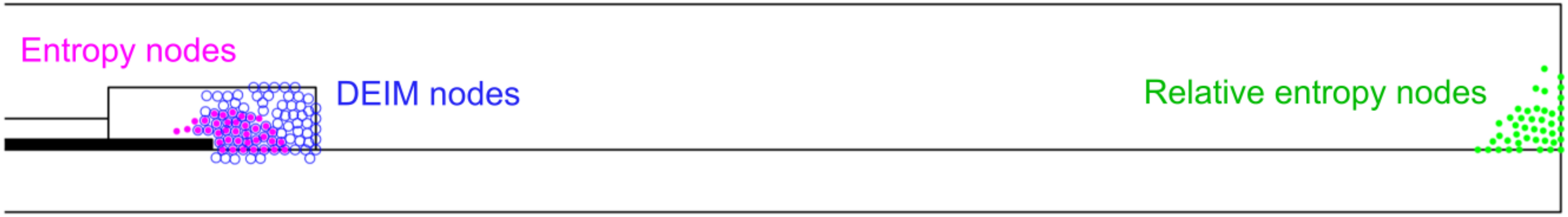
Maximal information refinement

- **Algorithm: Improved interpolation set, MIR method**
  - **Find nodes $\mathbb{H}$ that are high in Shannon entropy H**



  - **Find nodes $\mathbb{K}$ that are high in mean per node D_{KL} - highly discriminated**

  - **Get DEIM nodes $\mathbb{D}$**
  - **Interpolate nonlinear functions using constraints at nodes $\mathbb{H} \cup \mathbb{K} \cup \mathbb{D}$**



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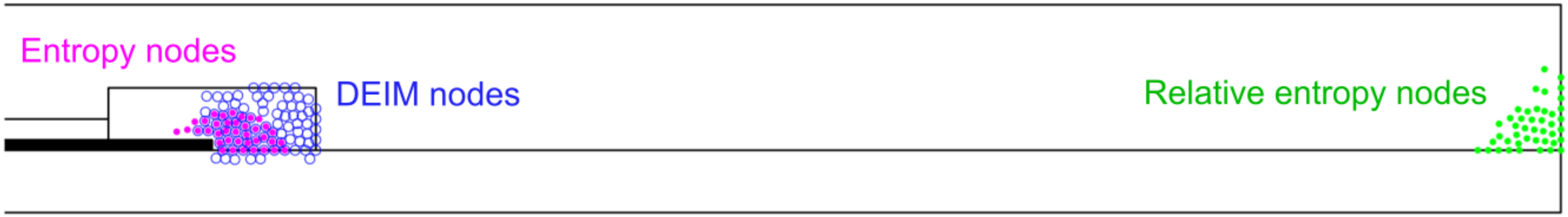
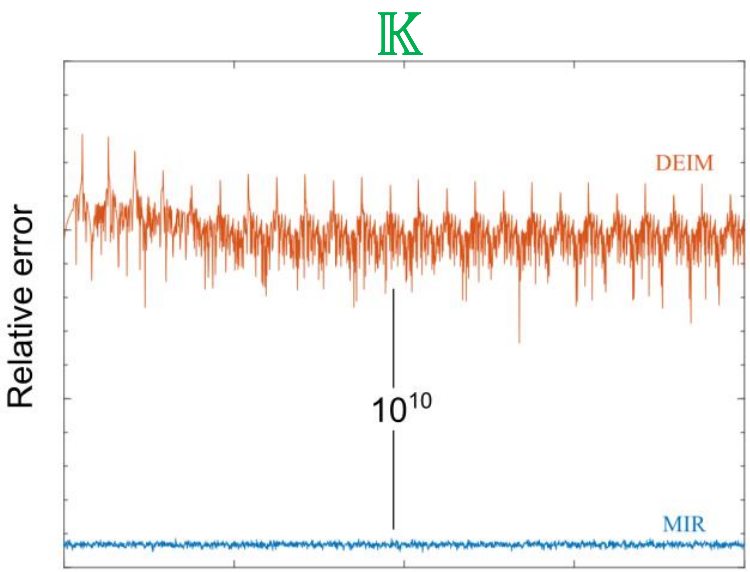
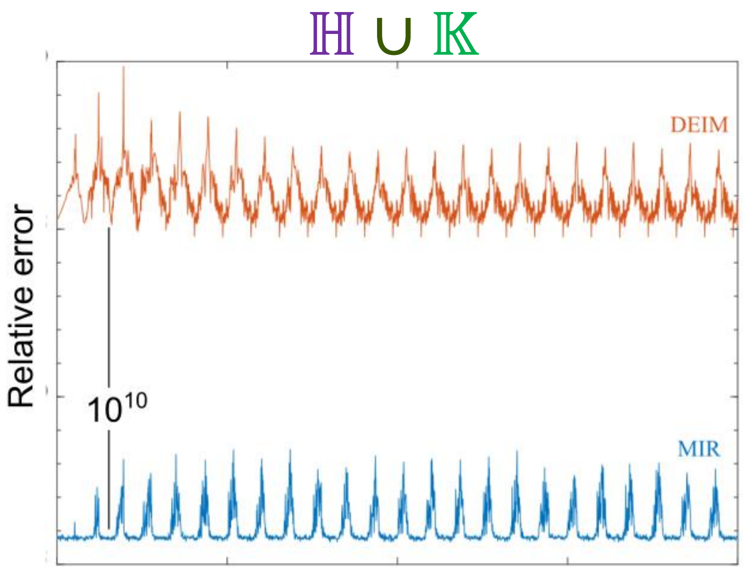
Maximal information refinement

- **Example: cable termination model with varistor**







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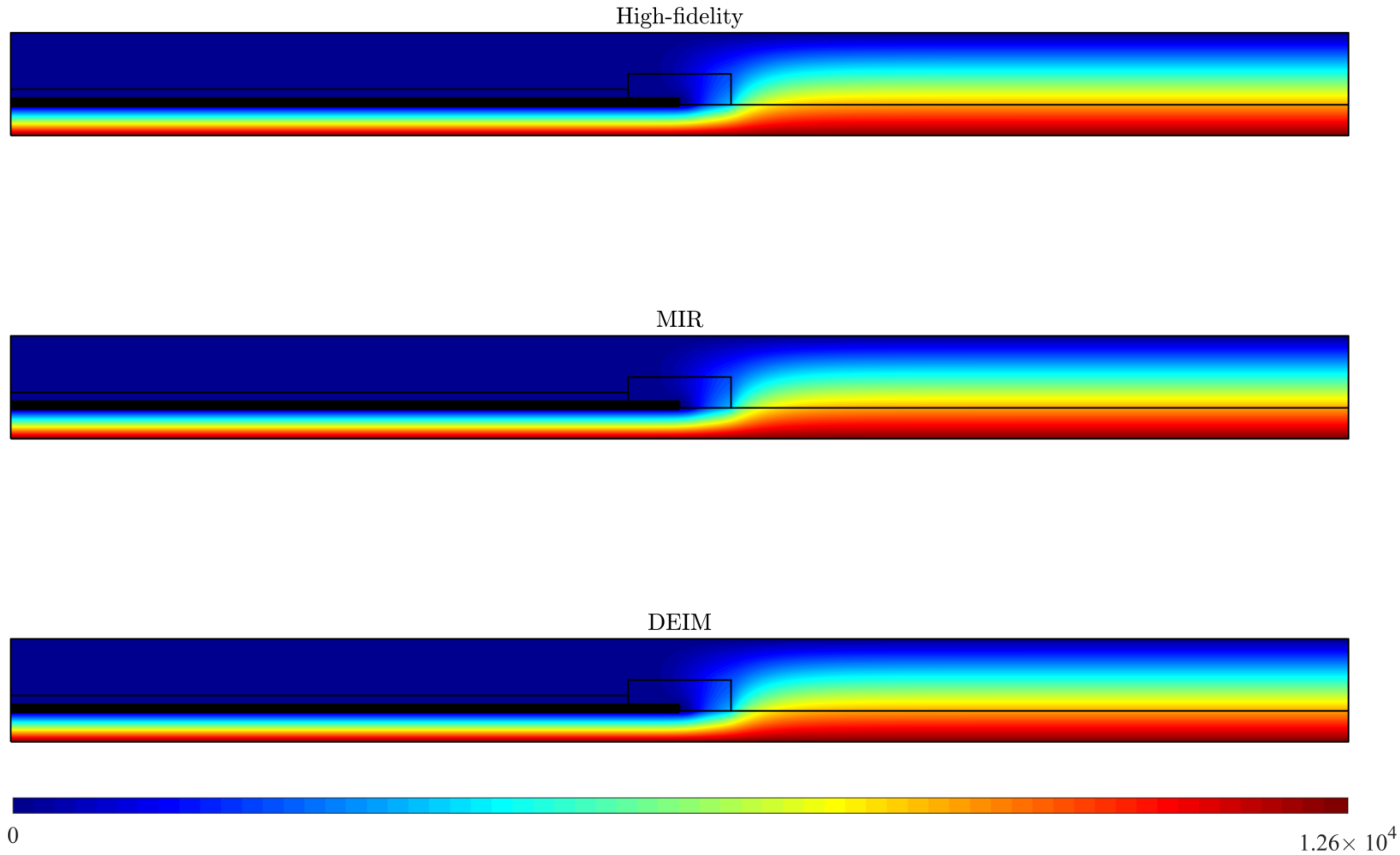
Maximal information refinement

- **Example: potential at $t = 0.12 sec$ throughout model**



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Discussion

- **Black-box parallel implementation in Python/FEniCS**
- **Large electro-quasistatics problem testing**
- **Development of mathematical framework (on its way)**
- **Analysis-based improvements to completely remove the greedy algorithm**

Thank you for your attention!

Questions?