Modeling and simulation of eddy current testing of aeronautical tubes

General context

The aircraft elements work under various constraints (thermal, mechanical, etc)

Produced effects

Cracks, creep, thermal fatigue and corrosion

These defects can cause serious damages

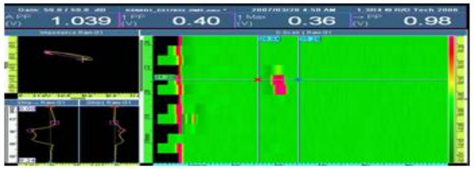
So

Assessing the damage of materials is a key point to control durability and reliability of parts and materials in service.

The Non destructive testing (NDT) gathers the most widespread methods for detecting defects of a part or review the integrity of a structure.

Objectives

Simulation and modeling



To detect and identify the position and dimension of the defect in structures or assemblies

NDT by electromagnetic methods

Mathematical and numerical formulation

The fundamental equations of electromagnetism are the Maxwell's equations, which arise in the following forms:

Mr. Maxwell

Equation of *conservative flow*:	$\nabla B = 0$
Equation of *Maxwell-Faraday*:	$\nabla E = -\partial B/\partial T$
Equation of *Maxwell-Gauss*:	$\nabla E = \rho/\varepsilon$
Equation of *Maxwell-Ampere*:	$\nabla B = \mu\ (J + \varepsilon\ dB/dt)$

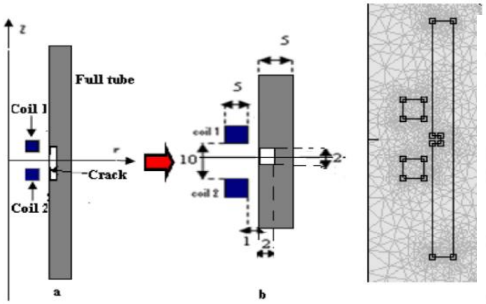
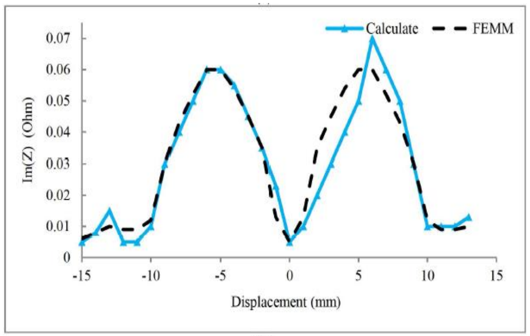


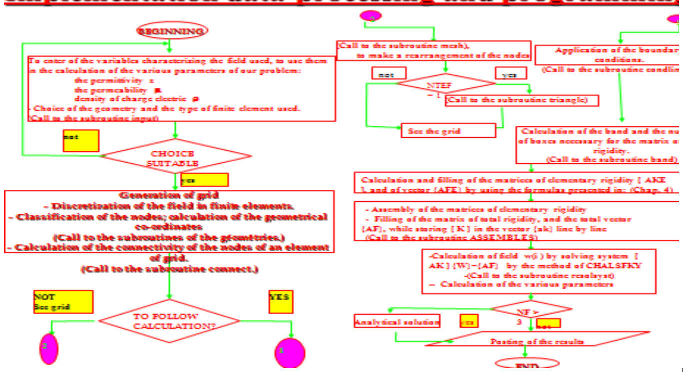
Figure 1: Problem description. (a): studied configuration, (b): geometrical dimensions (given in millimeters).



(b)

Figure 3: Impedance variation according to the current of 5 Khz. (a) real components and (b) imaginary components

Implementation data-processing and programming



CONCLUSION

The development of new probes and techniques for rapid and accurate inspection of tubes is of considerable interest to various industries.

The NDT by eddy currents is largely employed to inspect conducting materials. In this context, the tools for simulation enable to study the interactions (probe-part) and play an important role to conceive the systems of control and to show their performances.

The NDT–EC problems and Maxwell's equations allow to obtaining the evolution of the electric and magnetic fields in the continuous field.

The objective of this work is to use the finite element method to modeling the NDT–EC to provide a theoretical and computational framework for the efficient and approximate treatment of this electromagnetic problem.

The resolution of the problem is performed by FEM method where the magnetic potential vector A is found. After the electromagnetic field distribution analysis, the electromagnetic energy is calculated.

From the calculated energy, we deduce the real and imaginary components of the probe impedance that enable to determine the existence and effects of a crack under various frequencies applied on aluminum tubes.

The code validation is made with results given by FEMM code. The obtained results converge quickly towards the solution given by FEMM code with an average error of 0.018 for real parts of impedance and 0.004 for imaginary parts. That enables to use the code to carry out simulations for various similar cases with different input data. The code carried out allows the calculation of the eddy current field in various cases, the magnetic flux and the probe responses.

The suggested methodology can be applied to various probes and for various materials.

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Mathematical and numerical formulation

The differential equations representing the two-dimensional problems in extreme cases are given by the following general form:

$$-\frac{\partial}{\partial x}\left(\alpha_x \frac{\partial \Phi}{\partial x}\right)-\frac{\partial}{\partial y}\left(\alpha_y \frac{\partial \Phi}{\partial y}\right)+\beta\Phi = f$$

Concerning the boundary conditions, which can be presented, they are given by:

$\Phi_0 = p$ out of Γ_1

$$\left(\alpha_X \frac{\partial \Phi}{\partial x}\hat{x}\right)+\left(\alpha_y \frac{\partial \Phi}{\partial y}\hat{y}\right)\hat{n}+\gamma\Phi=q$$

•Variational formulation by the method of GALERKIN:

- After the development of the equation, we use the formulation of ***GALERKIN***.

•The residual one of the equation (II) is given by:

$$r = -\frac{\partial}{\partial x}\left(\alpha_x \frac{\partial \Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\alpha_y \frac{\partial \Phi}{\partial y}\right)+\beta\Phi - f \qquad \textbf{(III)}$$

•The residual one balanced is given by: $R_i^e = \iint_{\Omega_e} N_i^e \, r \, \partial x \partial y$ **(IV)**

•Let us substitute (III) in (IV), we find:

$$R_i^e = \iint_{\Omega_e} N_i^e \left[-\frac{\partial}{\partial x}\left(\alpha_x \frac{\partial \Phi}{\partial x}\right)-\frac{\partial}{\partial y}\left(\alpha_y \frac{\partial \Phi}{\partial y}\right)+\beta\Phi - f\right]\partial x \partial y \qquad \textbf{(V)}$$

- Let us use the similar identification given by the formulation and the theorem of divergence, we obtain (VI):

$$R_i^e = \iint_{\Omega_e}\left(\alpha_x \frac{\partial N_i^e}{\partial x}\frac{\partial \Phi}{\partial x}+\alpha_y \frac{\partial N_i^e}{\partial y}\frac{\partial \Phi}{\partial y}+\beta N_i^e \Phi\right)\partial x \partial y - \iint_{\Omega_e} N_i^e f \partial x \partial y - \int_{\Gamma_e} N_i^e D \tilde{n} \partial \Gamma$$

To obtain the elementary equation of each element, we must introduce the elementary approximate function:

$$\Phi^e(x,y) = \sum_{j=1}^{nn} N_i^e(x,y)\Phi_j^e \qquad \textbf{(VII)}$$

Thus let us substitute (VII) in (VI), we obtain:

$$R_i^e = \sum_{j=1}^{nn}\iint_{\Omega_e}\Phi_j^e\left(\alpha_x \frac{\partial N_i^e}{\partial x}\frac{\partial N_j^e}{\partial x}+\alpha_y \frac{\partial N_i^e}{\partial y}\frac{\partial N_j^e}{\partial y}+\beta N_i^e \partial N_j^e\right)\partial x \partial y - \iint_{\Omega_e} N_i^e f \partial x \partial y - \int_{\Gamma_e} N_i^e D \tilde{n} \partial \Gamma$$

It is the equation (VIII), given in matrix form:

$$\{R_E\} = [K_E].\{\Phi_E\} - \{B_E\} - \{G_E\} \qquad \textbf{(IX)}$$

Finally we summon our elementary systems to obtain the total system:

$$\{R\} = \sum_{e=1}^{ne}\{R_E\} = ([K_E].\{\Phi_E\} - \{B_E\} - \{G_E\}) = 0. \qquad \textbf{(X)}$$

That we can write :$[k].\{\Phi\} = \{B\} + \{G\}$.