

NASCA2018

Numerical Analysis and Scientific
Computation with Applications

Book of Abstracts

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June 27, 2018



National and Kapodistrian University of Athens

Welcome note from the Rector

Dear participants of NASCA 18 Conference,

It is a great pleasure to extend a warm welcome to you all attending the International Conference on Numerical Analysis and Scientific Computation with Applications (NASCA 18), organized by the Department of Mathematics of the National and Kapodistrian University of Athens in collaboration with the University of the Littoral - Opal Coast (ULCO), France.

The National and Kapodistrian University of Athens (NKUA), which recently celebrated 180 years since its foundation, is the oldest University in Greece and the first Higher Education Institution in the Balkan and Eastern Mediterranean area.

As Rector of this historical University, I fully support these initiatives which bridge the worlds of education and pursuit research. I strongly believe that participation in a Conference broadens our minds and gives us the opportunity to share and interchange our knowledge and research with colleagues from other countries, following the maxim “The best way to keep our power is to share it”.

Just as the decimals of the number π are infinite, so too are the experiences you can have in life. Your participation in NASCA18 will strengthen your international collaborations, will contribute in the progress of your scientific research and will offer you the unique experience of being in Greece in July and feel our wonderful summer atmosphere.

Concluding, I would like to congratulate the organizing committee of the conference and cordially wish you all, a fruitful conference.

The Rector

Meletios-Athanasios Dimopoulos
Professor of Medicine

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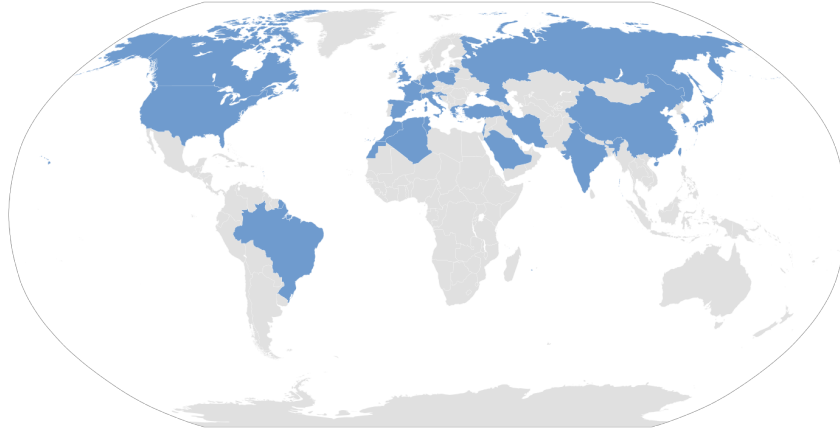
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Numerical Analysis and Scientific Computation with Applications
(NASCA18)

NASCA18 is the third International Conference on Numerical Analysis and Scientific Computation with Applications, which is organized by the National and Kapodistrian University of Athens (NKUA) and the University of the Littoral - Opal Coast (ULCO), in Kalamata, Greece, July 2-6, 2018. The present book of abstracts includes 10 invited lectures, 2 sponsored presentations, 79 contributed talks and 6 posters, presented by more than 100 participants originating from 27 countries which are depicted below.



The main scope of the conference is to bring together diverse research and practitioners from academic, research laboratories, and industries to present and discuss their recent works on numerical analysis and scientific computation with industrial applications. Topics include

- Large Linear Systems and Eigenvalue Problems with Preconditioning,
- Linear Algebra and Control, Model Reduction,
- Ill-posed Problems, Regularizations,
- Numerical Methods for PDEs,
- Approximation Theory, Radial Basis Functions, Meshless Approximation,
- Optimization,
- Applications to Image and Signal Processing, Environment, Energy Minimization, Internet Search Engines.

Refereed papers will be published in a special issue of Journal of Computational and Applied Mathematics.

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- **Dario Bini**, University of Pisa, Italy
- **Paul Van Dooren**, Catholic University of Louvain, Belgium
- **Vassilios Dougalis**, National and Kapodistrian University of Athens and FORTH-IACM, Greece
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- **Hassane Sadok**, LMPA, Université du Littoral Côte d'Opale, France
- **Michael Tsatsomeros***, Washington State University, USA

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Sessions of the conference

- A** Numerical PDEs
- B** Numerical linear algebra
- C** Regularization methods
- D** Data Assimilation
- E** ODEs, PDEs & Integral Equations
- F** Control theory
- G** Linear algebra
- H** Inverse problems
- I** Optimisation
- J** Approximation theory
- K** Image processing
- L** Model reduction

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Invited Talks

Generalized Matrix Functions: Theory and Computation

Michele Benzi¹

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Abstract

Generalized matrix functions were introduced in 1973 by Hawkins and Ben-Israel [4] with the aim of extending the notion of matrix function to rectangular matrices. If A has rank r and $A = U_r \Sigma_r V_r^*$ is a compact SVD of A , then $f^\diamond(A) := U_r f(\Sigma_r) V_r^*$ defines a generalized matrix function of A , provided that the real-valued scalar function f is defined on the singular values of A . Generalized matrix functions arise naturally in several applications, ranging from the solution of rank-constrained matrix minimization problems to the analysis of directed networks. When A is large, computing its SVD becomes prohibitively expensive. Hence, approximation methods are required for computing quantities related to generalized matrix functions, such as $f^\diamond(A)\mathbf{v}$ for a given vector \mathbf{v} . This talk will discuss theoretical aspects of generalized matrix functions, such as the preservation of structural properties present in A [3], as well as approximation methods based on Golub–Kahan bidiagonalization [1] and on Chebyshev polynomial interpolation [2].

Collaborators on this project include Francesca Arrigo, Caterina Fenu, Ru Huang, Jared Aurentz, Anthony Austin, and Vassilis Kalantzis.

References

- [1] F. Arrigo, M. Benzi, and C. Fenu, *Computation of generalized matrix functions*, SIAM Journal on Matrix Analysis and Applications, 37 (2016), 836–860.
- [2] J. L. Aurentz, A. P. Austin, M. Benzi, and V. Kalantzis, *Stable computation of generalized matrix functions via polynomial interpolation*, Preprint, December 2017. Submitted.
- [3] M. Benzi and R. Huang, *Structural properties preserved by generalized matrix functions*, in preparation.
- [4] J. B. HAWKINS AND A. BEN-ISRAEL, *On generalized matrix functions*, Linear and Multilinear Algebra, 1 (1973), pp. 163–171.

Quasi-Toeplitz matrices: analysis, algorithms and applications

Dario A. Bini¹

¹Università di Pisa

Abstract

Let $a(z) = \sum_{i=-\infty}^{+\infty} a_i z^i$ be a complex valued function defined for $z \in \mathbb{C}, |z| = 1$. The semi-infinite matrix $T(a) = (t_{i,j})_{i,j \in \mathbb{Z}^+}$ is said *Toeplitz matrix* associated with $a(z)$ if $t_{i,j} = a_{j-i}$. Typically, Toeplitz matrices are encountered in mathematical models where a shift invariance property, in time or in space, is satisfied by some entity.

Many queueing models from the applications are described by *quasi-Toeplitz* (QT) matrices, that is, matrices of the form $A = T(a) + E$ where E is a compact correction. For instance, in the random walk along a half-line, the probability transition matrix is the sum of a semi-infinite tridiagonal Toeplitz matrix and a correction E which is nonzero only in the entry (1,1). More complex situations are encountered if the domain of the random walk is the quarter plane where the probability matrix is block Toeplitz with Toeplitz blocks plus finite rank corrections.

The main computational problems that one encounters in this framework include computing matrix functions, solving polynomial matrix equations, and solving linear systems where the input matrices are QT.

In this talk, after pointing out the role of QT matrices in certain applications, we introduce some matrix norms, which make the class of QT matrices a Banach algebra and at the same time, are computationally tractable. Then we introduce the class of QT matrices representable by a finite number of parameters together with a matrix arithmetic on this class. This way, we may approximate QT matrices by using a finite number of parameters in the same way as real numbers are approximated by floating point numbers.

Finally, we introduce algorithms for the solution of the main computational problems encountered in this framework like computing the inverse matrix by means of the Wiener-Hopf factorization, computing the matrix exponential, solving quadratic matrix equations encountered in Quasi-Birth-Death stochastic processes where matrix coefficients are QT matrices.

Examples of applications are shown together with numerical experiments.

Numerical Analysis of long-wave models for surface water waves

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Abstract

In this talk, attention will be given to long-wave (shallow water) models that describe two-way propagation of surface water waves, approximating the 2d Euler equations. These will include the nonlinear hyperbolic system of the shallow water equations, the weakly nonlinear dispersive Boussinesq systems, the fully nonlinear dispersive Serre (or Green-Naghdi) equations, and the Camassa-Holm equation. A survey will be given of issues such as modelling, well-posedness of the various models, and the numerical analysis of these systems. Results of numerical experiments that illuminate properties of solitary-wave solutions of the dispersive systems will also be shown.

RandNLA: Randomization in Numerical Linear Algebra

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Abstract

The introduction of randomization in the design and analysis of algorithms for matrix computations (such as matrix multiplication, least-squares regression, the Singular Value Decomposition (SVD), etc.) over the past 15 years provided a new paradigm and a complementary perspective to traditional numerical linear algebra approaches. These novel approaches were motivated by technological developments in many areas of scientific research that permit the automatic generation of large data sets, which are often modeled as matrices.

In this talk, we will outline how such approaches can be used to approximately solve problems such as least-squares and ridge-regression problems or approximate the Singular Value Decomposition (SVD) of matrices. Applications of the proposed algorithms to data analysis tasks (with a particular focus in population genetics) will also be discussed.

Error estimate for the Gauss quadrature formula: The Gauss-Kronrod vs the anti-Gaussian approach

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Abstract

It is well known that a practical error estimator for the Gauss quadrature formula is by means of the corresponding Gauss-Kronrod quadrature formula developed by Kronrod in 1964. However, recent advances show that Gauss-Kronrod formulae fail to exist, with real and distinct nodes in the interval of integration and positive weights, for several of the classical measures. An alternative to the Gauss-Kronrod formula, as error estimator for the Gauss formula, is the anti-Gaussian and the averaged Gaussian quadrature formulae presented by Laurie in 1996. These formulae always exist and enjoy the nice properties that, in several cases, Gauss-Kronrod formulae fail to satisfy. After a brief overview of the Gauss-Kronrod, the anti-Gaussian and the averaged Gaussian formulae, we try to answer the question whether there are measures for which the Gauss-Kronrod and the averaged Gaussian formulae coincide, thus leading to the same error estimate for the Gauss formula. It is quite remarkable that this is true for a certain, quite broad, class of measures, which is described in terms of the three-term recurrence relation that the corresponding orthogonal polynomials satisfy.

The Arnoldi process for ill-posed problems

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Abstract

The Arnoldi process is the basis for the GMRES method, which is one of the most popular iterative methods for the solution of large linear systems of algebraic equations that stem from the discretization of a linear well-posed problem. The Arnoldi process and GMRES also can be applied to the solution of ill-posed problems. This talk discusses properties of Tikhonov regularization and iterative methods, that are based on the Arnoldi process, for the solution of linear ill-posed problems. The talk presents joint work with Silvia Gazzola, Silvia Noschese, Paolo Novati, and Ronny Ramlau.

Dimension reduction techniques: Algorithms and applications

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Abstract

A common tool that is exploited in solving data mining and machine learning problems is that of ‘dimension reduction’. Dimension reduction is based on the precept that the observed data often lies in a noisy version of a low-dimensional subspace and so it is critical to work in this subspace not only to reduce computational cost but also to improve accuracy. The talk will start with an overview of the key concepts and then illustrate dimension reduction methods with applications such as information retrieval, face recognition and matrix completion for recommender systems. One of the main difficulties in many of the methods based on dimension reduction is to find the inherent approximate rank of the data at hand. We will show how a few simple random sampling methods for computing spectral densities and counting eigenvalues can be used for this purpose. Finally, if time permits, we will report on our first experiments in ‘materials informatics’, a methodology which blends data mining and materials science.

Review of the convergence of some Krylov subspaces methods for solving linear systems of equations with one or several right hand sides

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Abstract

Krylov subspace methods are widely used for the iterative solution of a large variety of linear systems of equations with one or several right hand sides.

In this talk, we will derive new bounds for the GMRES method of Saad and Schultz, for solving linear system. We will give a sample formula for the norm of the residual of GMRES based on the eigenvalue decomposition of the matrix and the right hand side. This formula allows us to generalize the well known result on the convergence behavior of GMRES when the matrix has a full set of eigenvectors. The explicit formula of the residual norm of the GMRES when applied to normal matrix, which is a rational function, is given in terms of eigenvalues and of the components of the eigenvector decomposition of the initial residual. By minimizing this rational function over a convex subset, we obtain the sharp bound of the residual norm of the GMRES method applied to normal matrix, even if the spectrum contains complex eigenvalues. Hence we completely characterize the worst case GMRES for normal matrices. We use techniques from constrained optimization rather than solving the classical min-max problem (problem in polynomial approximation theory)

Known as one of the best iterative methods for solving symmetric positive definite linear systems, CG generates as FOM an Hessenberg matrix which is symmetric then triangular. This specific structure may be really helpful to understand how does behave the convergence of the conjugate gradient method and its study gives an interesting alternative to Chebyshev polynomials. The talk will deals also about some new bounds on residual norms and error A -norms using essentially the condition number. We will show how to derive a bound of the A - norm of the error by solving a constrained optimization problem using Lagrange multipliers.

References

- [1] M. BELLALIJ AND H. SADOK, *Approximation and convex programming approaches for solving a minmax problem*, Elect. Trans. Num. Anal., 33 (2008), pp. 17-30.
- [2] I. C. F. IPSEN, *Expressions and bounds for the GMRES residuals*, BIT, 40 (2000), pp.524–535.
- [3] J. LIESEN AND P. TICHY, *The worst-case GMRES for normal matrices*, BIT, 44 (2004), pp. 79–98.
- [4] Y. SAAD, *Iterative Methods for Solving Sparse Linear Systems*, SIAM, Philadelphia, 2003.
- [5] Y. SAAD AND M.H. SCHULTZ, *GMRES : a Generalized Minimal Residual algorithm for solving nonsymmetric linear systems*, SIAM J. Sci. Stat. Comput., 7 (1986), pp.856–869.
- [6] H. SADOK, *Analysis of the convergence of the Minimal and the Orthogonal residual methods*, Numer. Algorithms, 40 (2005), pp. 101-115.

Envelope: Localization for the Spectrum of a Matrix

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Abstract

New and old results will be presented on the *envelope*, $E(A)$, which is a bounded region in the complex plane that contains the eigenvalues of a complex matrix A . $E(A)$ is the intersection of an infinite number of regions defined by elliptic curves. As such, $E(A)$ resembles and is contained in the numerical range of A , which is the intersection of an infinite number of half-planes. The *envelope*, however, can be much smaller than the numerical range, while not being much harder to compute.

References

- [1] A. Aretaki, P.J. Psarrakos, and M.J. Tsatsomeros. The Envelope of Tridiagonal Toeplitz Matrices and Block-Shift Matrices. *Linear Algebra and its Applications*, 532:60-85, 2017.
- [2] M. Adam and M. Tsatsomeros. An Eigenvalue Inequality and Spectrum Localization for Complex Matrices. *Electronic Journal of Linear Algebra*, 15:239-250, 2006.
- [3] P.J. Psarrakos and M.J. Tsatsomeros. An envelope for the spectrum of a matrix. *Central European Journal of Mathematics*, 10:292-302, 2012.
- [4] P.J. Psarrakos and M.J. Tsatsomeros. On the geometry of the envelope of a matrix. *Applied Mathematics and Computation*, 244:132-141, 2014.

Linearizations of polynomial and rational matrices

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Abstract

We show that the problem of linearizations of polynomial and rational matrices is closely related to the polynomial matrix quadruples introduced by Rosenbrock in the seventies to represent rational transfer functions of dynamical systems. We also recall the concepts of irreducible and strongly irreducible quadruples which were introduced in the eighties, and show how they relate to the linearizations that are more common in the numerical linear algebra community. We then show that the family of strong linearizations of matrix polynomials, called “block Kronecker pencils”, as well as their extension to rational eigenvalue problems, nicely fit in that general framework. The novelty of these block Kronecker pencils is that they can be proven to be backward stable in a structured sense, for the polynomial matrix case as well as for the rational matrix case.

This is based on joint work with F. Dopico (UC3M), P. Lawrence (KULeuven), J. Perez (UMontana) and M.C. Quintana (UC3M).

Sponsored Talks

Application of PLSQ to Special Function Values

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Abstract

We are demonstrating how advanced semi-numerical techniques help in reconstructing exact values from numerical evaluation of special functions and definite integrals of such functions. Among the considered special functions are integrals of Airy wave functions and special values of auxiliary elliptic functions that arise as quotients of Jacobi Theta series. Applied techniques include LLL (Lenstra-Lenstra-Lovasz method of finding “short” vectors in integer lattices) and PSLQ (Partial Sum of Least Squares, by Bailey and Ferguson). Both techniques are implemented in Maple’s IntegerRelations package, and wrapped into the “identify” command for ease of usage. However, these techniques typically require very high precision and hence substantial computing power. Several examples show the successful application of PSLQ method in particular. Exact results returned by these methods must not be mixed up with proven results, but can serve as a starting point for further investigation.

Keywords: PSLQ, LLL, Special Functions, Minimal Polynomials, Maple, “identify” command

New Interpolation features in Maple 2018

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Abstract

Maple 2018 contains a new package for interpolating structured and unstructured data in any dimension. The supported methods include Kriging, inverse distance weighted, lowest / nearest / natural neighbor, radial basis functions, and more. The Interpolate command provides an interface to all interpolation methods. It directly returns objects that behave like normal mathematical functions. Previous versions of Maple already included interpolation methods for 1-dimensional data and data given for a grid of points. In Maple 2018, it is possible to interpolate data given for points in arbitrary, unstructured locations. The kriging interpolation method supports some extra functionality backed by statistical theory. In particular, it allows one to generate random data that is spatially correlated according to a so-called variogram. We will demonstrate interactive examples of all new interpolation methods in Maple 2018.

Keywords: Interpolation, Irregular Meshing, Structured and Unstructured Data, Maple

Contributed talks

A.

Numerical PDEs

A Stabilized bi-grid method for both Allen-Cahn and Navier-Stokes equations

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Abstract

In this talk, we propose a bi-grid scheme framework for both Allen-Cahn and Navier-Stokes equations in Finite Element Method. The new techniques are based on the use of two finite element spaces, a coarse one and a fine one, and on a decomposition of the solution into mean and fluctuant parts. This separation of the scales, in both space and frequency, allows to build a stabilization on the high modes components: the main computational effort is concentrated on the coarse space on which an implicit scheme is used while the fluctuant components of the fine space are updated with a simple semi-implicit scheme; they are smoothed without deteriorating the consistency. A coupling for both equations is made and the numerical examples we give show the good stability and the robustness of the new method. An important reduction of the computation time is also obtained when comparing our methods with fully implicit ones.

References

- [1] H. Abboud, C. Al Kosseifi, J.-P. Chehab. *A Stabilized bi-grid method for Allen Cahn equation in Finite Elements*. Submitted
- [2] H. Abboud, V. Girault, T. Sayah. *A second order accuracy for a full discretized time-dependent Navier-Stokes equations by a two-grid scheme*. Numer. Math. DOI 10. 1007/s00211-009-0251-5.
- [3] S. M. Allen, J. W. Cahn. *A microscopic theory for antiphase boundary motion and its application to antiphase domain coarsening*. Acta Metall. Mater., 27:10851095, 1979.
- [4] M. Brachet, J.-P. Chehab. *Stabilized Times Schemes for High Accurate Finite Differences Solutions of Non-linear Parabolic Equations*. Journal of Scientific Computing, 69 (3), 946-982, 2016
- [5] B. Costa, L. Dettori, D. Gottlieb, R. Temam. *Time marching techniques for the nonlinear Galerkin method*. SIAM J. SC. comp., 23, (2001), 1, 46–65.

Finite difference Eulerian-Lagrangian schemes for hyperbolic problems with discontinuous flux and stiff source

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Abstract

We formally develop a family of finite-difference shock-capturing schemes. This work also considers the questions of convergence of finite-difference approximations towards the entropic weak solution (correct shocks) of scalar, one-dimensional conservation laws with strictly convex and nonconvex flux functions. The finite-difference scheme is extended towards the viscosity solution of scalar, nonlinear multi-dimensional nonconvex (e.g., Buckley-Leverett) and convex (e.g., inviscid Burgers) model problems, which are presented and discussed. A new feature of the proposed method is the tracing forward to deal with balance laws and hyperbolic problems instead of trace backward in time over each *time step interval*. Indeed, we do not use approximate/exact Riemann solvers, and we do not use upwind source term discretizations either. Thus, we have a simple and fast Lagrangian-Eulerian solver for hyperbolic problems with discontinuous flux and stiff source. Our approach is based on a space-time Eulerian-Lagrangian framework introduced in [1]. Numerical tests show the robustness and accuracy of the method for a wide range of non-trivial applications available in the literature [2, 3, 4].

References

- [1] E. Abreu, W. Lambert, J. Perez and A. Santo, A new finite volume approach for transport models and related applications with balancing source terms, *Mathematics and Computers in Simulation* 137 (2017) 2-28.
- [2] F. Coquel, J-M Hérard, K. Saleh and N. Seguin, A robust entropy-satisfying finite volume scheme for the isentropic Baer-Nunziato model, *ESAIM: Mathematical Modelling and Numerical Analysis* 48(1) (2014) 165-206.
- [3] L. Gosse, *Computing Qualitatively Correct Approximations of Balance Laws Exponential-Fit, Well-Balanced and Asymptotic-Preserving*, SIMAI Springer (2013).
- [4] J. M. Greenberg, A. Y. LeRoux, R. Baraille, and A. Noussair, Analysis and Approximation of Conservation Laws with Source Terms, *SIAM J. Numer. Anal.* 34(5) (1997) 1980-2007.

MPDATA Meets Black-Scholes: Derivative Pricing as a Transport Problem

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Abstract

MPDATA stands for Multidimensional Positive Definite Advection Transport Algorithm. The iterative, explicit-in-time algorithm was introduced in [1] as a robust numerical scheme for atmospheric modelling applications. Extensions and generalisations of MPDATA continuously developed over the years constitute a family of numerical schemes offering high-order, sign-preserving and non-oscillatory solutions for transport problems (for a review, see e.g. [2], recent developments include third-order accurate formulation [3]). There is a multitude of documented applications of MPDATA across diverse domains. In the present work we demonstrate applicability of the algorithm for solving PDEs arising in financial derivative instrument pricing.

We present a generalisable framework for solving Black-Scholes-type equations by first transforming them into advection-diffusion problems, and numerically integrating using an iterative explicit finite-difference approach, in which the Fickian term is represented as an additional advective term. Leveraging this mathematical equivalence between Black-Scholes-type models and transport models, we detail applications of MPDATA to numerically reproduce the analytical solution of a celebrated benchmark problem — the Black-Scholes formula for pricing of European options — and to numerically solve the associated free boundary problem arising in the valuation of American options. These results are used for convergence analysis.

Presented work is based on [4]. Numerical solutions are obtained using libmpdata++ [5].

References

- [1] Smolarkiewicz, P.K. (1984), A fully multidimensional positive definite advection transport algorithm with small implicit diffusion, *J. Comp. Phys.* 54, doi:10.1016/0021-9991(84)90121-9
- [2] Smolarkiewicz, P.K. (2006), Multidimensional positive definite advection transport algorithm: an overview, *Int. J. Num. Meth. Fluids* 50, doi:10.1002/fld.1071
- [3] Waruszewski, M., Kühlein, C., Pawlowska, H. and Smolarkiewicz, P.K. (2018), MPDATA: Third-order accuracy for variable flows, *J. Comp. Phys.* (in press), doi:10.1016/j.jcp.2018.01.005
- [4] Arabas, S. and Farhat, A. (2017), MPDATA Meets Black-Scholes: Derivative Pricing as a Transport Problem, arXiv:1607.01751
- [5] Jaruga, A., Arabas, S., Jarecka, D., Pawlowska, H., Smolarkiewicz, P.K. and Waruszewski, M. (2015), libmpdata++ 1.0: a library of parallel MPDATA solvers for systems of generalised transport equations, *Geosci. Model Dev.*, 8, doi:10.5194/gmd-8-1005-2015

Regularization and differential quadrature procedures for dynamic analysis of beams with arbitrary discontinuities

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Abstract

Many physical and mechanical phenomena that can be well described by means of the Dirac-delta function and its derivatives. For instance, the dynamics of beams with an arbitrary number of Kelvin-Voigt viscoelastic rotational joints, translational supports, and attached lumped masses under heat source points can be mathematically modeled by means of the Dirac-delta function and its derivatives. The resulting partial differential equation has to be handled by means of well adapted numerical procedures. In this work, the differential quadrature method (DQM) is adapted for space an implicit scheme for time discretisation. The DQM is a straightforward method that can be implemented with few grid points and resulting with a reasonably good accuracy. However, the DQM is well-known to have some difficulty when applied to partial differential equations involving singular functions like the Dirac-delta function. This is caused by the fact that the Dirac-delta function and its derivatives cannot be directly discretized by the DQM. To overcome this difficulty, this work presents a combination of the DQM with a regularization procedure. Thanks to this regularization of the Dirac-delta and its used derivatives, the resulting differential equations can be directly discretized using the DQM. To validate the applicability of the proposed formulation and its implementation, computational examples of beams with arbitrary discontinuities are considered. The obtained results are well compared with the analytical and numerical results available in the literature.

References

- [1] S. DI LORENZO, C. ADAM, A. BURLON, G. FAILLA, A. PIRROTTA, *Flexural vibrations of discontinuous layered elastically bonded beams*; Journal of Composites Part B: Engineering, vol. 135, pp. 175-188, 2018.
- [2] Y. Belkourchia, L. Azrar, and E. M. Zeriab, *Hybrid optimization procedure and application to location optimization of piezoelectric actuators and sensors for active vibration control*; Journal of Applied Mathematical Modelling, 2018, accepted.
- [3] D. K. Hoffman, and D. J. Kouri, *Distributed approximating function theory: a general, fully quantal approach to wave propagation*; Journal of Physical Chemistry Physics, vol. 3, pp. 1179-1184, 1992.

Solving nonlinear systems of PDEs with the Partition of Unity - RBF method via the trust-region algorithm.

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Abstract

It is well known that globally-supported Radial Basis Function (RBF) collocation schemes have wonderful approximation properties of partial differential equations (PDEs), but are restricted to rather smallish problems. The recent Partition of Unity - RBF (PURBF) method [1] substantially lifts this limitation, while upholding the geometrical flexibility, straightforward coding, intuitive discretisation, and spectral accuracy of the global RBF approach. Moreover, when combined with the RBF-QR algorithm [2], the notorious trade-off between stability and accuracy also stops being a delicate issue. These features lend PURBF the potential of achieving the breakthrough of RBF methods into large-scale PDE applications.

In this work, we extend the trust-region scheme for nonlinear elliptic PDEs introduced in [3] to PURBF.

As a relevant application, we solve the steady flow of a viscous fluid past a cylinder, arguably the simplest—yet not trivial—example of fluid-structure interaction. As the Reynold's number grows, the eddy structure in the wake of the flow becomes increasingly involved. In order to adequately resolve it, the discretization must be refined in the recirculation area, while the computational domain must be extended further away from it.

We pose the relevant Navier-Stokes equations in natural variables and discretize them with the PURBF method. The resulting nonlinear system of collocation equations is optimally handled with the trust-region algorithm. The accuracy of the proposed numerical method is demonstrated by comparison with the reference solution [4] (up to $Re = 40$), and by monitoring the drag coefficient, beyond it.

References

- [1] A. Safdari-Vaighani, A. Heryudono and E. Larsson: A radial basis function partition of unity collocation method for convection-diffusion equations arising in financial applications. *J. Sci. Comput.* **64**, 341–367 (2015).
- [2] B. Fornberg, E. Larsson, and N. Flyer: Stable computations with Gaussian radial basis functions. *SIAM J. Sci. Comput.* **33**, 869–892 (2011).
- [3] Bernal, F.: Trust-region methods for nonlinear elliptic equations with radial basis functions. *Comput. Math. Appl.* **72**(7), 1743–1763 (2016).
- [4] R. Gautier, D. Biau, and E. Lamballais: A reference solution of the flow over a circular cylinder at $Re = 40$. *Computers & Fluids* **75**, 103–111 (2013).

Numerical approximation to the fractional non-linear Schrödinger equation

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Abstract

In this talk, a numerical method for the fractional nonlinear Schrödinger equation (fNLS) (see e. g. [1, 3, 4] and references therein) will be introduced and analyzed. The scheme will be applied to study the dynamics of traveling soliton solutions of the cubic fNLS whose existence was recently derived in [2].

References

- [1] Hong, Y., Sire, Y., On the fractional Schrödinger equation in Sobolev spaces *Commun. Pure Appl. Anal.* 14, 2015, 22652282.
- [2] Hong, Y., Sire, Y., A new class of traveling solitons for cubic fractional nonlinear Schrödinger equations, *Nonlinearity*, 30, 2017, 1262-1286.
- [3] Klein, C., Sparber, C., Markowich, P., Numerical study of fractional nonlinear Schrödinger equations, *Proc. Roy. Soc. A* 470, 2014, 20140364.
- [4] Laskin, N., Fractional Schrödinger equation, *Phys. Rev. E.*, 66, 2002, 056108.

A robust Multi-Level Domain Decomposition Preconditioner for Reservoir Simulation scalable on Many-Core architecture

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Abstract

In the evolution of High Performance Computing, multi-core and many-core systems are a common feature of new hardware architectures. The introduction of very large number of cores at the processor level is really challenging because it requires to handle multi level parallelism at various levels either coarse or fine to fully take advantage of the offered computing power. The induced required programming efforts can be fixed with parallel programming models based on the data flow model and the task programming paradigm [1]. Nevertheless many of standard numerical algorithms must be revisited as they cannot be easily parallelized at the finest levels. Iterative linear solvers are a key part of petroleum reservoir simulation as they can represent up to 80% of the total computing time. In these algorithms, the standard preconditioning methods for large, sparse and unstructured matrices – such as Incomplete LU Factorization (ILU) or Algebraic Multigrid (AMG) – fail to scale on shared-memory architectures with large number of cores. Recently, multi-level domain decomposition (DDML) preconditioners [2], based on the popular Additive Schwarz Method (ASM), have been introduced. Their originality resides on an additional coarse space operator that ensures robustness and extensibility. Their convergence properties have been studied mainly for linear systems arising from the discretization of PDEs with Finite Element methods. In this paper, we propose an adaptation for reservoir simulations, where PDEs are usually discretized with cell centered finite volume schemes. We discuss on our implementation based on the task programming paradigm with a data flow model [3]. We validate our approach on linear systems extracted from realistic petroleum reservoir simulations. We study the robustness of our preconditioner with respect to the data heterogeneities of the study cases, the extensibility regarding the model sizes and the scalability of our implementation regarding the large number of cores provided by new KNL processors or multi-nodes clusters. Finally we benchmark this new preconditioner to the ILU0 and the AMG preconditioners, the most popular ones in reservoir simulation.

References

- [1] Gratien, Jean-Marc, An Abstract Object Oriented Runtime System for Heterogeneous Parallel Architecture, in proceedings of the 2013 IEEE 27th International Symposium on Parallel and Distributed Processing Workshops and PhD Forum, pp. 1203-1212, IEEE Computer Society, 2013
- [2] Dolean, Victorita and Jolivet, Pierre and Nataf, Frédéric, An introduction to domain decomposition methods, Algorithms, theory, and parallel implementation, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2015
- [3] Roussel, Adrien and Gratien, Jean-Marc and Gautier, Thierry, Using runtime systems tools to implement efficient preconditioners for heterogeneous architectures, Oil & Gas Science and Technology–Revue d’IFP Energies nouvelles, vol. 71, no. 6, p. 65, 2016

One approach to the numerical simulation of the filtration problem in the presence of wells with given total flow rates

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Abstract

First, we discuss an elliptic boundary-value problem, describing a filtration of single-phase liquid, in the presence of wells of relatively small diameters, on which the integral flow rates of the fluid are given at constant but unknown pressures. As it is known, the solution of such a problem is reduced to solving a set of auxiliary problems according to the number of wells. We propose an alternative approach that consists of solving only one problem in a mixed weak formulation. In this case, a mixed formulation in the form of a system of equations of the first order makes it possible to carry out the extension of the solution by a constant into the wells, and the approach can be treated as a sort of fictitious domain method. Numerical implementation is based on a mixed finite element method with the Raviart-Thomas basis functions of the least degree. Error estimates are obtained and the results of computational experiments are presented.

Second, we expand the approach proposed to the non-stationary problem of a two-phase liquid filtration, and to the 3D filtration problem with wells parallel to one of the coordinate axes. In this case, the total velocity and pressure satisfy the quasi-stationary system of saddle-type equations discussed above. The non-stationary equation for saturation is also obtained using the fictitious domain method. The main feature of the approach proposed is that pressure and saturation belong to the same functional space. This is achieved by setting the phase velocity as the orthogonal projection of the total velocity multiplied by the relative permeability of the phase to the subspace of vector-functions with square summable divergence. Similar arguments are given for approximation of finite-dimensional subspaces. In the case of 3D problem, the solution in the direction of the wells is much smoother, and, therefore, it makes sense to introduce fluxes only in the directions orthogonal to the wells. As a result, we arrive at a mixed weak formulation, which is anisotropic in the sense of smoothness. For the numerical implementation a combination of the finite element method and the mixed finite element method is used, which we call the anisotropic mixed finite element method.

Iterative methods for the obstacle problem of a Naghdi's shell

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Abstract

In this work we consider some iterative methods for solving a kind of variational inequalities arises when we considering the finite element approximation of the obstacle problem of a Naghdi's shell formulated in Cartesian coordinates. The solution of the variational inequality is sought in a convex and not necessarily linear subset and it must satisfy another constraint, namely, a tangency requirement on the rotation field. In order to handle these constraints we propose mixed formulation which leads to "systems" with double saddle point structure. Both Uzawa-type methods and preconditioned Krylov subspace methods are discussed.

References

- [1] F. Beik and M. Benzi, Iterative methods for double saddle point systems, (2017).
- [2] F. Ben Belgacem, C. Bernardi, A. Blouza and F. Taallah . On the Obstacle Problem for a Naghdi Shell. J Elast (2011) 103: 1-13

Numerical insights of an improved SPH method

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Abstract

In this paper we discuss on the enhancements in accuracy and computational demanding in approximating a function and its derivatives via Smoothed Particle Hydrodynamics. The standard method is widely used nowadays in various physics and engineering applications [1],[2],[3]. However it suffers of low approximation accuracy at boundaries or when scattered data distributions is considered. Here we reformulate the original method by means of the Taylor series expansion and by employing the kernel function and its derivatives as projection functions and integrating over the problem domain [3]. In this way, accurate estimates of the function and its derivatives are simultaneously provided and no lower order derivatives are inherent in approximating the higher order derivatives. Moreover, high order of accuracy can be obtained without changes on the kernel function avoiding to lead unphysical results such as negative density or negative energy that can lead to breakdown of the entire computation in simulating some problems [1]. The modified scheme obtains the required accuracy, but the high computational effort makes the procedure rather expensive and not easily approachable in the applications. To this aim we make use of fast summations to generate a more efficient procedure, allowing to tune the desired accuracy. Working with the Gaussian function we proceed by applying the improved fast Gaussian transform as valid alternative to efficiently compute the summations of the kernel and its derivatives [4]. We discuss about the accuracy and the computational demanding of the improved method dealing with different sets of data and bivariate functions.

References

- [1] G.R. Liu and M.B. Liu, Smoothed Particle Hydrodynamics - A Mesh-Free Particle Method, World Scientific Publishing, Singapore (2003).
- [2] M.B. Liu, and G.R. Liu, Smoothed particle hydrodynamics (SPH): An overview and recent developments, Archives of Computational Methods in Engineering, Vol. 17(1), pp. 25–76 (2010).
- [3] M.B. Liu, W.P. Xie and G.R. Liu, Restoring particle inconsistency in smoothed particle hydrodynamics, Appl. Numer. Math., Vol. 56(1), pp. 19–36 (2006).
- [4] V.I. Morariu, B.V. Srinivasan, V.C. Raykar, R. Duraiswami and L.S. Davis, Automatic online tuning for fast Gaussian summation, Advances in Neural Information Processing Systems (NIPS 2008), Vol. 21, pp. 1113–1120 (2009).

A Fast Local Relaxation Solver for Certain 4th Order PDEs

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Abstract

We consider the following abstract evolution problem

$$\begin{cases} u_t - \operatorname{div}(M(u)\nabla w) = 0, \\ w + E'_-(u) - E'_+(u) + \epsilon^2 \Delta u \in \partial I_S(u), \end{cases} \quad (1)$$

where $M, E_+, E_- : S \rightarrow \mathbb{R}$ with $M(u) \geq 0$ and E_\pm convex, $S \subset \mathbb{R}$ is convex (but not necessarily compact). The indicator function $I_S(u)$ is equal to 0 if $u \in S$ and ∞ otherwise, whereas the set-valued function $\partial I_S(u)$ is its subdifferential. Note in particular that (1) implies $u(x) \in S$ almost everywhere. A number of interesting problems can be cast in this framework:

- *Cahn-Hilliard*: $M(u) = 1$, $E_-(u) = \frac{u^2}{2}$, $E_+(u) = \frac{u^4}{4}$ and $S = \mathbb{R}$,
- *Thin films [1]*: $M(u) = \frac{u^3}{3}$, $E_-(u) = 0$, $E_+(u) = 0$ and $S = [0, \infty)$,
- *Deep quench obstacle problem [2]*: $M(u) = 1 - u^2$ (degenerate) or $M(u) = 1$ (non-degenerate), $E_-(u) = \frac{u^2}{2}$, $E_+(u) = 0$ and $S = [-1, 1]$.

Combining $E(u) = E_+(u) - E_-(u)$ (convex-concave splitting) allows us to associate a free energy $\int_\Omega E(u) \, dx + \frac{\epsilon^2}{2} \|\nabla u\|_{L^2}^2$ with each concrete problem. Our scheme is a reformulation of a semi-implicit FEM scheme introduced in [3] and based on an approximation introduced in [4]. We recast the scheme as a (discrete) convex optimization problem with convex constraints at each time step, and derive a set of relaxation operators that are guaranteed to preserve the total mass and the hard constraint $u \in S$, while reducing the free energy. Furthermore the operators are locally supported and can be applied in parallel, allowing for a highly efficient implementation on modern computer hardware (GPU acceleration). Finally, we discuss the connection of this scheme to a recent study of these type of problems as gradient flows in weighted-Wasserstein metrics [5].

References

- [1] Grün, G. and Rumpf, M., *Nonnegativity preserving convergent schemes for the thin film equation.*, Numerische Mathematik **87**, 113-152 (2000).
- [2] Bañas, L., Novick-Cohen, A. and Nürnberg, R., *The degenerate and non-degenerate deep quenching obstacle problem: A numerical comparison*, Networks and Heterogenous Media **8**, 37-64 (2013).
- [3] Bañas, L. and Nürnberg, R., *Phase field computations for surface diffusion and void electro-migration in \mathbb{R}^3* , Comput. Vis. Sci. **12**, 319-327 (2009).
- [4] Barrett, J.W., Blowey, J. F. and Garcke, H., *Finite element approximation of the Cahn-Hilliard equation with degenerate mobility*, SIAM J. Numer. Anal. **37**, 286-318 (1999).
- [5] Lisini, S., Matthes, D., Savaré, G., *Cahn-Hilliard and thin film equations with nonlinear mobility as gradient flows in weighted-Wasserstein metrics*, J. Differential Equations **253**, 814-850 (2014).

Optimal preconditioners for Fractional Differential Equations

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Abstract

Fractional partial order diffusion equations are used to describe diffusion phenomena, that cannot be appropriately modeled by the well known second order diffusion equations. We mention that Fractional Differential Equations (FDEs) are of numerical interest, since there exist only few cases in which the analytic solution is known. Using the implicit Euler formula and the shifted Grünwald formula, we lead to a linear system whose coefficient matrix has a Toeplitz like structure. Taking into account the spectral analysis of such kind of matrices ([1]), we propose a preconditioner for Krylov methods, that under some suitable assumptions performs superlinear convergence for such kind of systems. In addition we extend the idea of such a construction to cover the multidimensional case, i.e Fractional PDE's. A number of numerical examples show the effectiveness of the proposed preconditioners.

References

- [1] M. Donatelli, M. Mazza, S. Serra-Capizzano. *Spectral analysis and structure preserving preconditioners for fractional diffusion equations*. Journal of Computational Physics, 307 (2016) 262-279.

Kinetic-MHD numerical model of the interaction of an electron beam with the plasma

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Abstract

The research of the processes of a terahertz radiation generation is a topical for such physical problems as turbulence plasma heating on open magnetic traps, fast burning of a target in the inertial nuclear fusion and for other problems. The terahertz radiation can find implementation in the material investigation, production quality control, etc. There are many ways of obtaining terahertz radiation sources. One of such methods is getting the radiation by the interaction of an electron beam with the plasma. This process can be examined by both experiments and the numerical modeling. In the current work the numerical modeling which permits us to analyze the dependence of the radiation efficiency on different parameters has been carried out.

The most qualitative study can be carried out by a full kinetic numerical model. However the difficulties appearing at the practical implementation of such models deal with the big difference of characteristic space scales for electrons and ions. It leads to developing the hybrid (combined) models, where the kinetic Vlasov equation is used to describe the motion of one component of plasma and the magneto-hydrodynamic approach to describe the motion of another one. The decrease of requirements on the architecture and memory of computers, comparing with those of the fully kinetic models, provided the vast expansion of the hybrid models. The research based on such models is the most perspective with regard to a computational experiment.

In the present work a 3D hybrid numerical model to describe an electron plasma beam entering a plasma box surrounded by a vacuum is considered. Here to describe the motion of electrons of the beam and plasma the kinetic description in terms of guiding center is used, whether for ions the MHD approach is utilized. To solve the Vlasov equation the particle-in-cell (PIC) method is used. The comparison with a fully kinetic model will provide to determine the limits of use of the developed model and, also, will be an independent test to verify a 3D kinetic model.

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Coupled multipoint flux and multipoint stress mixed finite element methods for poroelasticity

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Abstract

We discuss mixed finite element approximations for the Biot system of poroelasticity. We employ a weak symmetry elasticity formulation with three fields - stress, displacement, and rotation. We study two elasticity formulations, with poroelastic and elastic stress as primary unknown, respectively. Stability bounds and error estimates are derived for both formulations for arbitrary order mixed spaces. We further develop a method that can be reduced to a cell-centered scheme for the displacement and the pressure, using the multipoint flux mixed finite element method for flow and the recently developed multipoint stress mixed finite element method for elasticity. The methods utilize the Brezzi-Douglas-Marini spaces for velocity and stress and a trapezoidal-type quadrature rule for integrals involving velocity, stress, and rotation, which allows for local flux, stress, and rotation elimination. We perform stability and error analysis and present numerical experiments illustrating the convergence of the method and its performance for modeling flows in deformable reservoirs.

Review and complements on dual mixed finite element methods for non-Newtonian fluid flow problems

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Abstract

For a steady and creeping flow of an incompressible quasi-Newtonian fluid, the most used formulation is based on the strain rate tensor. For Ω a bounded domain of \mathbb{R}^2 with a Lipschitz boundary Γ and a given mass forces \mathbf{f} defined in Ω , the combination of the conservation equations leads to the Nonlinear Stokes problem:

$$\begin{cases} -\operatorname{div} \left(2\nu(|\mathbf{d}(\mathbf{u})|) \mathbf{d}(\mathbf{u}) \right) + \nabla p = \mathbf{f} & \text{in } \Omega, \\ \operatorname{div} \mathbf{u} = 0 & \text{in } \Omega, \end{cases} \quad (1)$$

where \mathbf{u} and p , the unknowns of the problem, are the velocity and pressure, respectively. For $\nu_0 > 0$ a reference viscosity and r a fluid characteristic real parameter verifying $1 < r < \infty$, the viscosity function $\nu(\cdot)$, depending on $|\mathbf{d}(\mathbf{u})|$, is usually given by one of the two following famous models:

$$\forall x \in \mathbb{R}_+^*, \text{ Power law : } \nu(x) = \nu_0 x^{r-2}, \text{ Carreau law : } \nu(x) = \nu_0 \left(1 + x \right)^{(r-2)/2}.$$

System (1) is supplemented by a set of boundary conditions.

The generalized Stokes problem (1) and its approximation by standard finite elements was widely studied. In these works, only the primal variables velocity and pressure are taken into account. But, for various reasons, one may need also information on the dual variables as velocity gradient $\nabla \mathbf{u}$, strain rate tensor $\mathbf{d}(\mathbf{u})$, extra-stress tensor $\boldsymbol{\sigma} = 2\nu(|\mathbf{d}(\mathbf{u})|) \mathbf{d}(\mathbf{u})$ etc. To do so, one need to build appropriate mixed formulations. The aim of this work is to present an exhaustive review on the available techniques using the mixed formulations for the problem (1), obtained for example in [2, 3, 4], and give some new results on the approximation of those problems.

References

- [1] V.J. Ervin, J.S. Howell, I. Stanculescu. A dual-mixed approximation method for a three-field model of a nonlinear generalized Stokes problem, *Comput. Methods Appl. Mech. Engrg.* 197 (2008) 2886-2900.
- [2] M. Farhloul, A.M. Zine. A mixed finite element method for a quasi-Newtonian fluid flow, *Numer. Methods Partial Differential Equations* 20 (2004) 803-819.
- [3] M. Farhloul, A.M. Zine. A posteriori error estimation for a dual mixed finite element approximation of non-Newtonian fluid flow problems, *Int. J. Numer. Anal. Model.* 5 (2008) 320-330.
- [4] M. Farhloul, A. Zine. A dual-mixed finite element method for quasi-Newtonian flows whose viscosity obeys a power law and Carreau law, *Mathematics and Computers in Simulation* 141 (2017) 83-95.
- [5] G.N. Gatica, M. Gonzalez, S. Meddahi. A low-order mixed finite element method for a class of quasi-Newtonian Stokes flows. Part I: A priori error analysis, *Comput. Methods Appl. Mech. Engrg.* 193 (2004) 881-892.

B.
Numerical Linear Algebra

The rational-extended Krylov subspace method for model reductions

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Abstract

Consider the multi-input multi-output (MIMO) linear time-invariant (LTI) system described by the state-space equations

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector and $u(t), y(t) \in \mathbb{R}^p$ are the input and output vectors respectively of the (LTI) system (1). When working with high order models, it is reasonable to look for an approximate model:

$$\begin{cases} \dot{x}_m(t) &= A_m x_m(t) + B_m u(t) \\ y_m(t) &= C_m x_m(t), \end{cases}$$

such as $A_m \in \mathbb{R}^{m \times m}$, $B_m, C_m^T \in \mathbb{R}^{m \times p}$, $x_m(t), y_m(t) \in \mathbb{R}^m$, and $m \ll n$, while maintaining the most relevant properties of the original system (1).

Several approaches in this area have been used. Among these approaches are the Krylov subspace methods. These are projection methods that have played a major role in large scale model reductions. Projection-type methods determine an approximation of the approximate solution by projecting a given problem onto a much smaller approximation space. Projection-type methods determine an approximation of the approximate solution by projecting a given problem onto a much smaller approximation space. The approximation spaces that have been widely studied in the past for a variety of problems are the standard Krylov space, the inverse Krylov space, and the rational Krylov space. Each of the aforementioned spaces mentioned has advantages and disadvantages. Thereby, we introduce a new method that will be used for reducing the transfer function and can be extended to approach the solutions of Sylvester and Riccati equations.

The general idea of this method is to provide a new Krylov subspace that is richer than the rational Krylov subspace as well as the extended Krylov subspace. This idea comes from the lack of information on the matrix A when using rational Krylov subspace. That is why, we introduce a new method that we name the extended-rational Krylov method. The objective of this work is to exploit this new space to approach the dynamical system and the transfer function.

References

- [1] O. Abidi, M. Heyouni et K. Jbilou: The extended block and global Arnoldi methods for model reduction. Numerical Algorithms 75 (2017) 285-304.

Generalized Block Anti-Gauss Quadrature Rules

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Abstract

Golub and Meurant describe how pairs of Gauss and Gauss–Radau quadrature rules can be applied to determine inexpensively computable upper and lower bounds for certain real-valued matrix functionals defined by a symmetric matrix. However, there are many matrix functionals for which their technique is not guaranteed to furnish upper and lower bounds. In this situation, it may be possible to determine upper and lower bounds by evaluating pairs of Gauss and anti-Gauss rules. Unfortunately, it is difficult to ascertain whether the values determined by Gauss and anti-Gauss rules bracket the value of the given real-valued matrix functional. Therefore, generalizations of anti-Gauss rules have recently been described, such that pairs of Gauss and generalized anti-Gauss rules may determine upper and lower bounds for real-valued matrix functionals also when pairs of Gauss and (standard) anti-Gauss rules do not. The available generalization requires the matrix that defines the functional to be real and symmetric. The present paper extends generalized anti-Gauss rules in several ways: The real-valued matrix functional may be defined by a nonsymmetric matrix. Moreover, extensions that can be applied to matrix-valued functions are presented. Estimates of element-wise upper and lower bounds then are determined. Finally, modifications that yield simpler formulas are described.

A Householder-based algorithm for Hessenberg-triangular reduction

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Abstract

Reducing the matrix pair (A, B) to Hessenberg-triangular form is an important and time-consuming preprocessing step when computing eigenvalues and eigenvectors of the pencil $A - \lambda B$ by the QZ-algorithm. Current state-of-the-art algorithms for this reduction are based on Givens rotations, which limits the possibility of using efficient level 3 BLAS operations, as well as parallelization potential on modern CPUs. Both of these issues remain even with partial accumulation of Givens rotations [1], implemented, e.g., in LAPACK.

In this talk we present a novel approach for computing the Hessenberg-triangular reduction, which is based on using Householder reflectors. The key element in the new algorithm is the lesser known ability of Householder reflectors to zero-out elements in a matrix column even when applied from the right side of the matrix [2, 3]. The performance of the new reduction algorithm is boosted by blocking and other optimization techniques, all of which permit efficient use of level 3 BLAS operations. We also discuss measures necessary for ensuring numerical stability of the algorithm. While the development of a parallel version is future work, numerical experiments already show benefits of the Householder-based approach compared to Givens rotations in the multicore computing environment.

References

- [1] B. Kågström, D. Kressner, E. S. Quintana-Ortí, G. Quintana-Ortí, *Blocked algorithms for the reduction to Hessenberg-triangular form revisited*, BIT, 48(3):563–584, 2008.
- [2] D. S. Watkins, *Performance of the QZ algorithm in the presence of infinite eigenvalues*, SIAM J. Matrix Anal. Appl., 22(2):364–375, 2000.
- [3] B. Kågström, D. Kressner, *Multishift variants of the QZ algorithm with aggressive early deflation*, SIAM J. Matrix Anal. Appl., 29(1):199–227, 2006.

A rational QZ method

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Abstract

In this talk we introduce a rational QZ method for the dense, unsymmetric, generalized eigenvalue problem. The method operates on matrix pairs in Hessenberg form and implicitly performs nested subspace iteration driven by elementary rational functions. This is achieved without solving any systems. We review a direct reduction method to Hessenberg form and demonstrate that subspaces can be deflated already during the reduction phase. We introduce a shifted implicit rational QZ step on a Hessenberg pair. Numerical experiments demonstrate that a good choice of poles can significantly speedup the convergence compared to the QZ method.

References

- [1] Camps D., Meerbergen K., and Vandebril R., *A rational QZ method*, 2018, <https://arxiv.org/abs/1802.04094>.

Is Lanczos tridiagonalization essential for solving large eigenvalue problems?

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Abstract

In this talk we present a new type of restarted Krylov methods for calculating peripheral eigenvalues of a large sparse symmetric matrix, G . The new framework avoids the Lanczos tridiagonalization algorithm and the use of polynomial filtering. This simplifies the restarting mechanism and allows the introduction of several modifications. Convergence is assured by a monotonicity property that pushes the computed Ritz values toward their limits.

One improvement lies in the rule for determining the starting vector of the Krylov matrix that is built at each iteration. This rule is much simpler and intuitive than rules that are based on polynomial filtering. The Krylov matrix is generated by a three term recurrence relation that is based on MGS orthogonalization. These modifications lead to fast rate of convergence.

The differences between the new approach and the Lanczos approach become significant in inner-outer iterative methods which are using inexact inversions to compute small eigenvalues. The idea is to replace G with an approximate inverse of G , which is obtained by solving a linear system. So each column of the Krylov matrix requires the solution of a new linear system whose matrix is G . In this case Lanczos methods compute Ritz pairs of the approximate inverse of G , while our method computes Ritz pairs of G . In inner-outer methods most of the computation time is often spent on the inner iterations. This feature increases the appeal of the new approach, as the extra time that it pays for deserting the Lanczos algorithm becomes negligible.

The new approach is easily adapted to compute a partial SVD of a large sparse $m \times n$ matrix, A . Here it avoids the popular Lanczos bidiagonalization process. Instead the Krylov matrix that is built at each iteration is generated by using the cross-product matrix, ATA , or an approximate inverse of this matrix. Yet the singular triplets are computed from Ritz triplets of A . When using the cross-product matrix to compute low-rank approximations there is no need to update estimates of right singular vectors, which leads to considerable saving of storage.

The experiments that we have done are quite encouraging. In many cases the algorithm achieves the required accuracy within a remarkable small number of iterations.

One class learning as a null space kernel Rayleigh quotient. Application to abnormal events detection in video sequences

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Abstract

Many problems in machine learning can be simplified as the maximization (minimization) of a generalized Rayleigh quotient, given by:

$$\max_{\mathbf{w}} \rho(\mathbf{w}; Q, P) = \frac{\mathbf{w}^T Q \mathbf{w}}{\mathbf{w}^T P \mathbf{w}} \quad (1)$$

where Q and P are positive definite matrices, and $\mathbf{w} \neq 0$ is an optimal projection into a lower-dimensional space which solves the problem (1). Two classical applications formulated as (1) are principal component analysis (PCA) and linear discriminant analysis (LDA). In order to deal with non normal data distributions, a kernel embedding is commonly performed which generalizes the previous criterion for any kind of distributions. Recently, we proposed a new kernel Rayleigh quotient for solving the one class learning problem [1]. Unlike binary/multi-class classification methods, one class classification tries to isolate a target (positive) class when the negative class is either poorly represented, even not at all or is not well statistically defined. Our formulation introduces two regularized specific scatter matrices $Q(\mathbf{y})$ and $P(\mathbf{y})$ which are parameterized by an unknown binary vector \mathbf{y} determining the membership of a data to the positive or negative class. We show in [1] that the optimal separation of these two data populations amounts then to achieve two joint actions: *dimensionality reduction* and *classification*. However, several aspects limit its use: firstly, the accuracy of the dimensionality reduction depends on the representativeness of the abnormal data which can be low for specific data sets. Secondly, when the data dimensionality becomes much greater than the training sample size, the method can be badly conditioned (singularity problem).

In this paper, we introduce a null space based extension of this criterion. The principle of this extension is to introduce a joint subspace where the training target data set has zero covariance. Then, a simple distance measure can be derived in this subspace to decide about abnormality of a test data. We show here that this formulation implicitly avoids the singularity problem for under sampled data sets. We show also that the dimension of this specific subspace is directly linked to the contamination rate which is of course unknown. This issue is solved by introducing a single artificial counter-example. This operating strategy allows both to reduce the size of the null space to a single null direction which can be estimated from a simple power method and to maintain an enough generalization performance. A comparative study with different one class learning methods is conducted both on moderated and high dimensional data sets. An original application to the detection of abnormal events in video sequences is also presented.

References

- [1] F. Dufrenois and J.C. Noyer. One class proximal support vector machines. *Pattern Recognition*. 52, pp. 96–112, 2016.

A new eigenvalue algorithm for unitary Hessenberg matrices via quasiseparable representations

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Abstract

We present an algorithm to compute eigenvalues of a unitary Hessenberg matrix U . Such a matrix admits a quasiseparable of order one representation. We determine the eigenvalues of the matrix U via eigenvalues of the Hermitian matrix $A = \frac{1}{2}(U + U^*)$ and the anti-Hermitian matrix $B = \frac{1}{2}(U - U^*)$. The matrices A and B have quasiseparable of order two representations and we apply our previously developed methods to compute their eigenvalues.

A generalized matrix Krylov subspace method for TV regularization

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Abstract

In this work we present an efficient algorithm to solve total variation (TV) regularizations of images contaminated by a both blur and noise. The unconstrained structure of the problem suggests that one can solve a constrained optimization problem by transforming the original unconstrained minimization problem to an equivalent constrained minimization one. An augmented Lagrangian method is developed to handle the constraints when the model is given with matrix variables, and an alternating direction method (ADM) is used to iteratively find solutions. The solutions of some sub-problems are belonging to subspaces generated by application of successive orthogonal projections onto a class of generalized matrix Krylov subspaces of increasing dimension.

References

- [1] A. BOUHAMIDI AND K. JBILOU, *A note on the numerical approximate solution for generalized Sylvester Matrix equations*, Appl. Math. Comput., 206(2)(2008) 687694.
- [2] R. GLOWINSKI, *Numerical Methods for Nonlinear Variational Problems*. Springer Verlag, 2008
- [3] A. LANZA, S. MORIGI, L. REICHEL, AND F. SGALLARI, *A generalized Krylov subspace method for ℓ_p - ℓ_q minimization*. SIAM J. Sci. Comput. 37(5), S30S50 (2015)

Numerical Linear Algebra Aspects in the Analysis of Absorption Graphs

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Abstract

Graphs with absorption play an important role in applications such as the modeling of epidemics spreading. Recently, K. A. Jacobsen and J. Tien [1] have discussed properties of the *absorption inverse* L^d of the graph Laplacian L , a particular (1,2)-inverse of L , and have shown how the absorption inverse can provide a wealth of information on the structure of the underlying graph. For example, quantities associated with L^d can be used to define a distance on the graph, and to develop graph partitioning heuristics. Moreover, the row sums of L^d can be used to rank the nodes in a graph with absorption (i.e., they provide a centrality index). In this talk we will discuss some computational aspects of the absorption inverse, including the use of matrix factorization and of iterative methods for computing L^d and quantities associated with it. Furthermore, we examine alternative centrality measures for ranking the nodes of graphs with absorption and compare them to the one proposed by Jacobsen and Thien.

References

- [1] K. Jacobsen, J. Tien, A generalized inverse for graphs with absorption, *Linear Algebra and its Applications*, 537 (2018), pp. 118–147.

On a block approach for approximating selected elements of the matrix inverse

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Abstract

Approximating selected elements of the matrix inverse and more general matrix functions is a topic of importance in several applications, e.g. in data analytics [3]. The challenge is to achieve the approximation more efficiently than the obvious alternative, that is computing the matrix function in full. Recent methods to this effect, e.g. [4], attempt to exploit structural characteristics such as element decay away from the main diagonal. We present a method for achieving the approximation of blocks of the matrix inverse lying along the diagonal. Inspired by the probing techniques proposed in [4] we generalize to the case of block diagonally dominant matrices and exploit the resulting norm-decay of the blocks of the inverse [1][2]. We present numerical experiments that illustrate the features and performance of our method and its application.

References

- [1] C.Echeverría, J. Liesen and R. Nabben. Block diagonal dominance of matrices revisited: bounds for the norms of inverses and eigenvalue inclusion sets. arXiv.org, December 2017.
- [2] D. G. Feingold and R. S. Varga Block diagonally dominant matrices and generalizations of the Gerschgorin circle theorem, *Pacific J. Math.*, 12:1241–1250, 1962
- [3] V. Kalantzis, C. Bekas, A. Curioni, E. Gallopoulos. Accelerating data uncertainty quantification by solving linear systems with multiple right-hand sides. *Numerical Algorithms*, 62(4):637-653, January 2013.
- [4] J. M. Tang and Y. Saad. A probing method for computing the diagonal of a matrix inverse. *Numerical Linear Algebra with Applications*, 19(3):485-501, 2012

Approximate solutions to large nonsymmetric differential Riccati problems with applications to transport theory

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Abstract

In this talk, we will be considering the nonsymmetric differential Riccati matrix equation (NDRE in short) of the form

$$\begin{cases} \dot{X}(t) = -AX(t) - X(t)D + X(t)SX(t) + Q, & t \in [t_0; t_f] \\ X(t_0) = X_0, \end{cases} \quad (1)$$

where $A \in \mathbb{R}^{n \times n}$, $D \in \mathbb{R}^{p \times p}$, $Q \in \mathbb{R}^{n \times p}$, $S \in \mathbb{R}^{p \times n}$ and $X(t) \in \mathbb{R}^{n \times p}$.

The equilibrium solutions of (1) are the solutions of the corresponding nonsymmetric algebraic Riccati equation (NARE)

$$-AX - XD + XSX + Q = 0. \quad (2)$$

Large scale differential matrix equation are currently receiving a great deal of attention [4, 5]. For NDREs, there is no existing method in the large scale case to our knowledge. In this talk, we consider large scale NDREs with low rank right-hand sides. After some facts about the existence of exact solutions to (1), we will show how to apply the extended block Arnoldi algorithm to get low rank approximate solutions. We will consider the special case corresponding to NDREs from transport theory. Numerical experiments will show that this approach is promising for large-scale problems.

References

- [1] H. ABOU-KANDIL, G. FREILING, V. IONESCU, G. JANK, *Matrix Riccati Equations in Control and Systems Theory*, Systems & Control Foundations & Applications, Birkhauser, 2003.
- [2] D.A. BINI, B. IANNAZZO, F. POLONI, *A fast Newtons method for a nonsymmetric algebraic Riccati equation*, SIAM J. Matrix Anal. Appl., 30:276–290, 2008.
- [3] C-H. GUO, N. HIGHAM, *Iterative solution of a nonsymmetric algebraic Riccati equation* SIAM J. Matrix. Anal. Appl., 29(2):396–412, 2007.
- [4] M. HACHED, K. JBILOU, *Numerical solutions to large-scale differential Lyapunov matrix equations*, Numerical Algorithms, <https://doi.org/10.1007/s11075-017-0458-y>, 2017.
- [5] M. HACHED, K. JBILOU, *Computational Krylov based methods for large scale differential Sylvester matrix problems*, Numerical Linear Algebra with Applications, <https://doi.org/10.1007/s11075-017-0458-y>, 2018.

Absolute value circulant preconditioners for non-symmetric Toeplitz-related systems

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Abstract

Circulant preconditioning for symmetric Toeplitz systems has been well developed over the past few decades. For a large class of such systems, descriptive bounds on the convergence of the conjugate gradient method can be obtained. For nonsymmetric Toeplitz systems, most work had been focused on normalising the original systems until [J. Pestana and A. J. Wathen. SIAM J. MATRIX ANAL. APPL. Vol. 36, No. 1, pp. 273-288, 2015] recently showed that theoretic guarantees on the convergence of the minimal residual method can be established via a simple use of reordering. The authors further proved that a suitable absolute value circulant preconditioner can be used to ensure rapid convergence rate. In this talk, we show that the related ideas can also be applied to systems defined by functions of Toeplitz matrices. Numerical examples are given to support our results.

Estimating matrix functionals via extrapolation

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Abstract

A spectrum of applications arising from Statistics, Machine Learning, Network Analysis require the computation of matrix functionals of the form $x^T f(A)y$, where A is a diagonalizable matrix and x, y are given vectors. In this work we are interested in efficiently computing bilinear forms primarily due to their importance in several contexts. For large scale computation problems it is preferable to achieve approximations of the bilinear forms avoiding the explicit computation of the matrix function. For this purpose an extrapolation procedure has been developed, attaining the approximation of the bilinear form with one, two or three term estimates in a complexity of square order. The extrapolation procedure gives us the flexibility to define the moments of a matrix A either directly or through the polarization identity. The presented approach is characterized by easy applicable formulae of low complexity that can be implemented in vectorized form.

References

- [1] P. Fika, M. Mitrouli, Estimation of the bilinear form $y^* f(A)x$ for Hermitian matrices, *Linear Algebra Appl.*, 502, pp. 140-158, 2015.
- [2] P. Fika, M. Mitrouli, P. Roupas, Estimates for the bilinear form $x^T A^{-1}y$ with applications to linear algebra problems, *Elec. Trans. Numer. Anal.*, 43, pp.70-89, 2014.
- [3] P. Fika, M. Mitrouli, P. Roupas, Estimating the diagonal of matrix functions, *Math. Meth. Appl. Sci.*, 41, pp. 1083-1088, 2018.
- [4] G.H. Golub, G. Meurant: *Matrices, Moments and Quadrature with Applications*, Princeton University Press, Princeton, 2010.

Preconditioned GMRES Method for the Solution of Non-Symmetric Real Toeplitz Systems

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Abstract

It is well known that preconditioned conjugate gradient (PCG) methods are widely used to solve ill-conditioned real symmetric and positive definite Toeplitz linear systems $T_n(f)x = b$. This case has been entirely studied while the case of real non-symmetric and non-definite Toeplitz systems is still open. Toeplitz matrices have the same entries along their diagonals and are generated from the Fourier coefficients of a 2π -periodic generating function or symbol f . Such systems appear in various Mathematical Topics: Differential and Integral equations, Mechanics, Fluid Mechanics and in Applications: signal processing, image processing and restoration, time series, and queueing networks.

The PCG method, fails to solve non-symmetric and/or non-definite systems. In this paper we focus on finding fast and efficient methods, based on Krylov subspaces, to solve large real non-symmetric Toeplitz systems. Especially, we concentrate on the solution of such systems using the Preconditioned Generalized Minimum Residual (PGMRES) method. Real non-symmetric Toeplitz systems are generated by symbols f being complex 2π -periodic functions of the form $f = f_1 + if_2$, where f_1 is a 2π -periodic even function while f_2 is a 2π -periodic odd one. If f_1 and f_2 have roots at some points in $[-\pi, \pi]$, then the problem becomes ill-conditioned and some kind of preconditioning is necessary. We propose efficient band Toeplitz preconditioners to solve this kind of problems. The preconditioners are generated by symbols being trigonometric polynomials, which aim at raising the roots and on the other hand at giving some kind of approximation to the functions f_1 and f_2 . We achieve a good clustering of the singular values of the preconditioned matrix in a small interval around 1. Also, we discuss on how to generalize the results of the unilevel case to Block Toeplitz systems (2-level), which are generated by 2-variate complex functions.

Finally, we show the efficiency of the proposed technique by presenting various numerical experiments.

A generalized global Arnoldi method based on tensor format for ill-posed tensor equations*

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Abstract

We study feasibility of using a method, obtained by developing the generalized global Arnoldi process in conjunction with Tikhonov regularization, based on tensor format (called GGAT_BTF method) to solve ill-posed tensor equations. More precisely, with the aid of the results given in [Numer. Linear Algebra Appl. 23 (2016) 444–466] and strategies used in [J Comput. Appl. Math. 236 (2012) 2078–2089], we construct an algorithm to solve Sylvester tensor equations with severely ill-conditioned and possibly full coefficient matrices. Some theoretical results are presented and applicability of the proposed is numerically examined for image restoration and solving a Sylvester tensor equation arising from exploiting the Chebyshev collocation spectral method to solve the 3D radiative transfer equation (RTE).

Keywords: Generalized Arnoldi process, Sylvester tensor equation, ill-conditioned, regularization

References

- [1] Bader BW and Kolda TG. MATLAB Tensor Toolbox Version 2.5. <http://www.sandia.gov/tgkolda/TensorToolbox>.
- [2] Beik FPA, Movahed FS and Ahmadi-Asl S. On the Krylov subspace methods based on tensor format for positive definite Sylvester tensor equations. *Numer. Linear Algebra Appl.* 2016; **23**: 444–466.
- [3] Bouhamidi A, Jbilou K, Reichel L and Sadok H. A generalized global Arnoldi method for ill-posed matrix equations. *J Comput. Appl. Math.* 2012; **236**: 2078–2089.
- [4] Reichel L, Sgallari F and Qiang Ye Q. Tikhonov regularization based on generalized Krylov subspace methods. *Appl. Numer. Math.* 2012; **62**: 1215–1228.
- [5] Ya-Song S, Jing M and Ben-Wen L. Chebyshev collocation spectral method for three-dimensional transient coupled radiative–conductive Heat transfer. *Journal of Heat Transfer.* 2012; **134**: 092701–092707.

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Efficient calculating the selected eigenvalues of parametrized matrices

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Abstract

In applications of linear algebra including nuclear physics and structural dynamics, there is a need to deal with uncertainty in the matrices. We focus on matrices that depend on a set of parameters ω . In this talk, we are interested in the smallest eigenvalue of a large scale generalized eigenvalue problem with symmetric positive definite matrices since in that case the eigenvalues are real and the eigenvectors satisfy some orthogonality properties. If ω can be interpreted as the realisation of random variables, one may be interested in statistical moments of the smallest eigenvalue. In order to obtain statistical moments, we need a fast evaluation of the eigenvalue as a function of ω . Since calculating this is costly for large matrices, we are looking for a small parametrized eigenvalue problem, whose smallest eigenvalue makes a small error with the smallest eigenvalue of the large eigenvalue problem.

The advantage, in comparison with a global polynomial approximation (on which, e.g., the polynomial chaos approximation relies), is that we do not suffer from the possible non-smoothness of the smallest eigenvalue. The small scale eigenvalue problem is obtained by projection of the large scale problem. The idea is to filter out the subspace which is not needed for determining the smallest eigenvalue. We developed projection methods based on the principle that an eigenvalue of the projected eigenvalue problem is also an eigenvalue of the large eigenvalue if the eigenvector is present in the associated subspace. By this we interpolate between the points for which the eigenvector are added to the subspace.

Besides this, we will also talk about the choice of interpolation points, how to efficiently check the overall error and compare our method with a polynomial approximation. Numerical examples from structural dynamics are given.

Treating breakdowns and near breakdowns in JHESS-algorithm for a reducing a matrix to upper J -Hessenberg form

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Abstract

The reduction of a matrix to an upper J -Hessenberg is usually performed via the algorithm JHESS (or via the recent algorithm JHMSH and its variants). This reduction is a crucial step in the SR -algorithm (which is a QR -like algorithm), structure-preserving, for computing eigenvalues and vectors, for a class of structured matrices. Unlike its equivalent in the Euclidean case, the JHESS-algorithm may meet a fatal breakdown, causing a brutal stop of the computations or encounter near-breakdowns, which are source of serious numerical instability.

In this talk, we present efficient strategies for curing fatal breakdowns and also for treating near breakdowns. The effectiveness of such strategies are illustrated by numerical experiments.

References

- [1] A. Bunse-Gerstner and V. Mehrmann, A symplectic QR-like algorithm for the solution of the real algebraic Riccati equation, IEEE Trans. Automat. Control **AC-31** (1986), 1104–1113.
- [2] L. Elsner, On some algebraic problems in connection with general eigenvalue algorithms, Linear Alg. Appl., 26:123-38 (1979).
- [3] A. Salam and H. Ben Kahla, An upper J -Hessenberg reduction of a matrix through symplectic Householder transformations, submitted.
- [4] A. Salam and H. Ben Kahla, A treatment of breakdowns and near breakdowns in a reduction of a matrix to upper J -Hessenberg form and related topics, submitted.
- [5] C. Van Loan, A symplectic method for approximating all the eigenvalues of a Hamiltonian matrix, Linear Algebra Appl. **61** (1984), 233–251.

Convergence of a Modified Newton Method for a Matrix Polynomial Equation Arising in Stochastic Problems

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Abstract

We consider the Newton iteration of a matrix polynomial equation

$$P(X) = \sum_{k=0}^n A_k X^k = A_n X^n + A_{n-1} X^{n-1} + \cdots + A_1 X + A_0 = 0, \quad (1)$$

which arises in stochastic problem. The elementwise minimal nonnegative solution S of (1) can be obtained using Newton's method with the zero initial value if the equation has the solution. Moreover, the convergence rate of the iteration is quadratic if P'_S , the Fréchet derivative at S , is nonsingular. If P'_S is singular, the convergence rate is at least linear. But, for any $\varepsilon > 0$, there exists an integer $i_0 > 0$ such that $\{X_i - S\}_{i=i_0}^\infty$ is in the ε -neighborhood of an one-dimensional space. Then, with a modified Newton method

$$X_{i+1} = X_i - \lambda P_{X_i}'^{-1}(P(X_i)), \quad (2)$$

we can reduce the iteration number. In the representation, we will give the proofs and some numerical experiments of this abstract.

References

- [1] Chun-Hua Guo, Nonsymmetric algebraic Riccati equations and Wiener-Hopf factorization for M -matrices. SIAM J. Matrix Anal. Appl., 23(1):225–242, 2001.
- [2] Chun-hua Guo and Peter Lancaster, Analysis and modification of Newtons method for algebraic Riccati equations. Math. Comp., 67(223):1089–1105, 1998.
- [3] Nicholas J. Higham and Hyun-Min Kim, Solving a quadratic matrix equation by Newtons method with exact line searches. SIAM J. Matrix Anal. Appl., 23:303–316, 2001.
- [4] C. T. Kelley, A shamanskii-like acceleration scheme for nonlinear equations at singular roots. Math. Comp., 47(176):609–623, 1986.
- [5] Jong-Hyeon Seo and Hyun-Min Kim, Convergence of pure and relaxed Newton methods for solving a matrix polynomial equation arising in stochastic models. Linear Algebra Appl., 440:34–49, 2014.

Jacobi-type Algorithm for Cosine-Sine Decomposition

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Abstract

Cosine-sine decomposition is defined for both, orthonormal matrices partitioned into 2×1 block matrix and for orthogonal matrices partitioned into 2×2 block matrix. We call these decompositions 2×1 CSD and 2×2 CSD, respectively. They display the connection between SVD's of the blocks of the matrix. Because of this connection, in the presence of small or close singular values, it is not trivial to compute decompositions accurately.

In this talk, we present a new method for computing the 2×1 CSD. The method runs two one-sided Jacobi SVD algorithms simultaneously, one for each block. These algorithms construct same sequence of transformations using information from both blocks. By doing so, the interconnection of two SVD's is maintained and the accuracy of the computed 2×1 CSD is achieved.

Additionally, we discuss how any accurate 2×1 CSD algorithm can be used to compute a 2×2 CSD.

Using the power of iterative methods for the SVD in machine learning

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Abstract

Machine learning has emerged as one of the primary clients for large scale singular value calculations. The matrices are typically very large so that dense SVD methods cannot be used. In computational sciences, iterative methods have been used to solve these type of problems, with methods such as the recently proposed Golub-Kahan-Davidson pushing the envelope in accuracy and convergence speed. In machine learning, combinations of randomized and iterative methods have risen drastically in popularity because of their extreme efficiency.

One important constraint in machine learning is the size and nature of the matrices. In some applications, matrices have no exploitable sparsity, which makes the matrix-vector multiplication expensive in terms of operations, and even more expensive in terms of memory accesses. In other applications, the matrix may only be available as streaming sets of columns or rows. However, the singular values often decay fast, giving rise to a near low rank structure.

Another important variable is how much of the singular space is needed and how accurately. Several applications require a small number (1-4) smallest or largest singular triplets to a good but not high accuracy. Others require the computation of a low rank approximation of the entire matrix. In these cases, low accuracy is sufficient, especially when the matrix itself is near low rank.

Given the size of the problems, one way to obtain a low rank approximation is to perform a randomized projection of the matrix down to a more manageable dimensionality where we can solve the SVD directly. The singular space approximations obtained by the randomized projection can be improved iteratively by applying a basic subspace iteration. These techniques have proven particularly efficient, requiring only 1-2 subspace iterations, and although the computed singular vectors are not accurate, the low rank approximation is. This has puzzled numerical analysts and only recently has research emerged that starts to explain the phenomenon.

In this talk, we examine the relative merits of these two classes of methods based on this recent research and some new observations. First, we observe that traditional block iterative methods are also randomized. Then we show that the difference between randomized and traditional iterative methods is only in their stopping criteria. Specifically, the requirement for a good low rank approximation is a weaker condition that does not depend on the singular gaps or even the angles between our approximations and the singular vectors. Therefore, a common framework is possible with flexible stopping criteria and variable block size for different problems.

References

- [1] S. Goldenberg, A. Stathopoulos, Copper Mountain Conference on Iterative methods, 2018
- [2] M. Gu, SIAM Journal on Scientific Computing 37,(3),2014
- [3] I. Ipsen and P. Drineas, arXiv:1801.00670, 2018

Computation of the matrix logarithm using the double exponential formula

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Abstract

We consider computation of the matrix logarithm based on the integral representation

$$\log(A) = \int_0^1 (A - I)[t(A - I) + I]^{-1} dt \quad (1)$$

where A is a square matrix whose eigenvalues do not lie on the closed negative real axis. The matrix logarithm arises in many applications such as von Neumann entropy and image registration.

The efficiency of computing the matrix logarithm of (1) by numerical integration depends on the choice of quadrature formula. The Gauss–Legendre quadrature is considered as one choice for (1) (e.g. [1]), because the Gauss–Legendre quadrature for (1) coincides with Padé approximation for $\log(A)$ (see, e.g. [2, p.274]). However, when the condition number of A is large, the Gauss–Legendre quadrature may not be the best choice.

In this talk, we consider the Double Exponential (DE) formula instead of Gauss–Legendre quadrature. As compared with the Gauss–Legendre quadrature, the DE formula usually works well even if integrands have endpoint singularities, and the DE formula improves accuracy at low cost (without recalculating abscissas-weight pairs). On the other hand, since the DE formula changes the integral interval into $(-\infty, \infty)$, we need to estimate the truncation error and give a promising finite interval for practical use. We propose an algorithm including the truncation error estimation, and show some numerical results to confirm its efficiency.

References

- [1] P. I. DAVIES AND N. J. HIGHAM, *Computing $f(A)b$ for matrix functions f* , in QCD and numerical analysis III, vol. 47 of Lect. Notes Comput. Sci. Eng., Springer, Berlin, 2005, pp. 15–24.
- [2] N. J. HIGHAM, *Functions of Matrices: Theory and Computation*, SIAM, Philadelphia, 2008.

Approximate Greatest Common Divisor through factorization of matrices of special structure

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Abstract

The Greatest Common Divisor (GCD) of polynomials is needed in many real applications such as Image Processing, Secret Sharing Schemes, Networks, Control Theory and has attracted the interest of researchers for many years. The computation of the degree and the coefficients of the GCD of polynomials is a hard task. Noise and measurement errors in initial data can lead to wrong results. Thus, it is of interest the relaxation of the notion of the exact GCD and the computation of an approximate GCD (AGCD) of polynomials. In this paper, we present methods for the computation of the degree and the coefficients of the AGCD of several polynomials. The approaches of this work are based on matrices of special structure such as Generalized Sylvester and Bezoutians. Classical methods such as LU, QR and RRQR factorization are used and appropriately modified in order to be applied to the special form of the matrices reducing the required computational complexity of the procedures. The error analysis of the methods guarantees the stability of the presented algorithms. Many numerical experiments testing and comparing the methods conclude to useful remarks.

References

- [1] T. Beelen and Van Dooren, P., A pencil approach for embedding a polynomial matrix into a unimodular matrix, *SIAM J. Matrix Anal. Appl.*, 9, 1988, pp. 77–89.
- [2] D. A. Bini and P. Boito, A Fast Algorithm for Approximate Polynomial GCD Based on Structured Matrix Computations, *Operator Theory: Advances and Applications*, 199, 2010, pp. 155–173.
- [3] R. M. Corless and S. M. Watt and L. Zhi, QR Factoring to compute the GCD of Univariate Approximate Polynomials, *IEEE Transactions on Signal Processing*, 52, No. 12, 2004, pp. 3394–3402.
- [4] G. M. Diaz-Toca and L. Gonzalez-Vega, Barnett’s theorems about the greatest common divisor of several univariate polynomials through Bezout-like matrices, *J. Symbolic Computation*, 34, 2002, pp.59–81.
- [5] N. Karcianas, Invariance properties and characterisation of the greatest common divisor of a set of polynomials, *Int. J. Control*, 46, 1987, pp. 1751–1760.
- [6] N. Karcianas and M. Mitrouli and D. Triantafyllou, Matrix Pencil methodologies for computing the greatest common divisor of polynomials: Hybrid algorithms and their performance, *Int. Journ. of Control*, 79, No. 11, 2006, pp. 1447–1461.
- [7] I. S. Pace and S. Barnett, Comparison of algorithms for calculation of g.c.d of polynomials, *Int. J. Control*, 4, No. 2, 1973, pp. 211–216.
- [8] T. Sasaki and M. Sasaki, Polynomial remainder sequence and approximate GCD, *ACM SIGSAM Bull.*, 31, 1997, pp. 4–10.
- [9] J. R. Winkler and X. Lao, The calculation of the degree of an approximate greatest common divisor of two polynomials, *J. Comp. Appl. Math.*, 235, 2011, pp. 1587–1603.

On the two-level iterative methods in the Krylov subspaces

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Abstract

We consider different approaches to constructing of the two - level iterative procedures in the Krylov subspace for a high-performance solution of the large systems of linear algebraic equations (SLAEs) with non-symmetric, in general, sparse matrices which arise in finite element or finite volume methods (FEM or FVB) for approximation of the multi-dimensional boundary value problems with real data on the non-structured grids, when implementating on the multi-processor computers (MPC) with distributed and hierarchical shared memory. The parallel algorithms are based on the additive domain decomposition methods (DDM) , with parametrized overlapping of subdomains and various interface conditions at the internal boundaries, in the Krylov subspaces. At the upper level, the block multi-preconditioned semi-conjugate direction methods (MP-SCD) are applied which realize semi-conjugate gradient or residual (MP-SCG or MP-SCR) algorithms in particular cases. These algorithms are equivalent, in a sense, to FOM and GMRES methods respectively. The acceleration of the external iterative process is provided by means of the coarse grid correction, or the deflation procedure, or the low - rank approximation of the original matrix, as well as by the Sonneveld approaches. In the case of reduced recursions, the acceleration of restarted iterations is attained by the least squares approaches. The simultaneous solutions of the auxiliary sub-systems in subdomains at the low -level of the computational process are implemented by means of either direct or iterative algorithms. The parallelization of the two - level iterative processes is based on the hybrid programming using MPI and multi-thread technologies. The performance and efficiency of the proposed algorithms, which are implemented in the library KRYLOV, are demonstrated on the results of numerical experiments for a representative set of test problems.

The Computation of the Greatest Common Divisor of Three Polynomials

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Abstract

This paper considers the computation of the greatest common divisor GCD of three polynomials, $f(y)$, $g(y)$ and $h(y)$, using the Sylvester resultant matrix and its subresultant matrices. It is shown that there are two variations of the subresultant matrices for this problem, and that they have 2×3 and 3×3 partitioned structures. The order of the polynomials for the 2×3 partitioned structure is important because each ordering yields subresultant matrices of different dimensions. Furthermore, the magnitudes of the entries of the matrices may differ significantly, and their numerical ranks are not consistent. It is therefore necessary to consider the optimal sequence of the subresultant matrices, and this issue is addressed. Also, it is shown that $f(y)$, $g(y)$ and $h(y)$ must be preprocessed before computations are performed on their Sylvester matrix and its subresultant matrices. Computational examples are presented and the singular values of the 2×3 and 3×3 forms, and the variation of the singular values between the different partitioned structures of the 2×3 forms, are shown. It is also shown that it is important to include the preprocessing operations because it yields an approximate GCD and coprime polynomials with much smaller errors than the errors obtained when the preprocessing operations are omitted.

C. Regularization Methods

An $\ell^p - \ell^q$ minimization method with cross-validation for the restoration of impulse noise contaminated images

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Abstract

Discrete ill-posed problems arise in many areas of science and engineering. Their solutions, if they exist, are very sensitive to perturbations in the data. Regularization aims to reduce this sensitivity. Many regularization methods replace the original problem with a minimization problem with a fidelity term and a regularization term. Recently, the use of a p -norm to measure the fidelity term and a q -norm to measure the regularization term has received considerable attention, see, e.g., [2, 3, 4] and references therein. The relative importance of these terms is determined by a regularization parameter. When the perturbation in the available data is made up of impulse noise and a sparse solution is desired, it is often beneficial to let $0 < p, q < 1$. Then the p - and q -norms are not norms, thus the minimized functional is non-convex. For the minimization of such non-convex functional we resort to the algorithm proposed in [4].

The choice of a suitable regularization parameter is crucial for the quality of the computed solution. It is therefore important to develop methods for determining this parameter automatically, without user-interaction. In this talk we discuss two approaches based on cross validation for determining the regularization parameter in this situation. Computed examples that illustrate the performance of these approaches when applied to the restoration of impulse noise contaminated images are presented.

References

- [1] A. Buccini and L. Reichel, An $\ell^p - \ell^q$ minimization method with cross-validation for the restoration of impulse noise contaminated images., Submitted, 2018.
- [2] R. H. Chan and H. X. Liang, Half-quadratic algorithm for ℓ_p - ℓ_q problems with applications to TV- ℓ_1 image restoration and compressive sensing, in Proceedings of Efficient Algorithms for Global Optimization Methods in Computer Vision, Lecture Notes in Comput. Sci. # 8293, Springer, Berlin, 2014, pp. 78–103.
- [3] C. Estatico, S. Gratton, F. Lenti, and D. Titley-Peloquin, A conjugate gradient like method for p -norm minimization in functional spaces, Numer. Math., 137 (2017), pp. 895–922.
- [4] G. Huang, A. Lanza, S. Morigi, L. Reichel, and F. Sgallari, Majorization-minimization generalized Krylov subspace methods for ℓ_p - ℓ_q optimization applied to image restoration, BIT Numer. Math., 57 (2017), pp. 351–378.

Acceleration of iterative regularization methods by delta-convex functionals in Banach spaces

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Abstract

We consider a linear functional equation $Ax = y$ where $A : X \rightarrow Y$ is an ill-posed operator between two complete normed linear functional spaces X and Y . The variational approach involves the minimization of a Tikhonov-type regularization functional $\Phi_\alpha : X \rightarrow \mathbb{R}$ defined as

$$\Phi_\alpha(x) = \frac{1}{p} \|Ax - y\|_Y^p + \alpha \mathcal{R}(x),$$

where $p > 1$, $\mathcal{R} : X \rightarrow \mathbb{R}$ is a convex penalty term which quantifies the “non-regularity” of x , and $\alpha > 0$ is the regularization parameter which balances between data fitting and stability.

In the simplest case, referred as (basic) Tikhonov regularization, both X and Y are Hilbert spaces, $p = 2$ and $\mathcal{R}(x) = \frac{1}{2} \|x\|_X^2$. In the last two decades, several extensions to Banach space settings have been proposed in the literature to reduce the over-smoothness effects of the basic Hilbertian approach, also aimed at improving the sparsity, or at enforcing non-negativity or other special constraints. We just mention some special weighted Lebesgue spaces L^p or Sobolev spaces $W^{k,p}$, with $1 < p < +\infty$ [4].

In this talk, we discuss a special Tikhonov-type functional Φ_α whose penalty term \mathcal{R} is model-dependent, that is, \mathcal{R} explicitly depends on the operator A which characterizes the functional equation [3]. In addition, the functional is no longer convex as in the conventional setting, but in our proposal is delta-convex; i.e., it is representable as a difference of two convex terms. We will show that the proposed delta-convex functional allows us to speed up the convergence of iterative gradient minimization algorithms. This acceleration technique, which we call as “irregularization”, is useful for large scale equations arising in image restoration [1]. Then, an extension of the algorithm to the unconventional variable exponent Lebesgue space $L^{p(\cdot)}$ [2] is also analyzed and numerically tested, aimed at providing a pointwise and adaptive control of the level of regularization.

References

- [1] Brianzi P., Di Benedetto F., Estatico, C., and Surace, L., 2017 Irregularization accelerates iterative regularization., *Calcolo*, under revision.
- [2] Diening, L., Harjulehto, P., Hästö, P., Ruzicka, M. 2011 *Lebesgue and Sobolev spaces with variable exponents*. Lecture Notes in Mathematics. vol. 2017.
- [3] Huckle, T. and Sedlacek M. 2012 Tykhonov-Phillips regularization with operator dependent seminorm. *Numer. Algor.*, vol. 60, pp. 339–353.
- [4] Schuster, T., Kaltenbacher, B., Hofmann, B., and Kazimierski, K. S. 2012 *Regularization Methods in Banach Spaces*. Radon Series on Computational and Applied Mathematics, vol. 10, De Gruyter.

Numerical methods for estimating the tuning parameter in penalized least squares problems

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Abstract

The solution of the penalized least squares problems depends on a tuning parameter. A popular tool for specifying the tuning parameter is the generalized cross-validation (GCV). In this work, we are concerned with the estimation and minimization of the GCV function by using a combination of an extrapolation procedure and a statistical approach. We apply simulations for different statistical designs and we report the Type I and Type II error rates in order to compare the behaviour of the proposed method with the corresponding estimates of the tuning parameter which are obtained by minimizing the exact GCV function. The Type I and Type II error rates are computed considering the L_1 , the hard thresholding and the Smoothly Clipped Absolute Deviation penalty functions.

References

- [1] E. Androulakis, C. Koukouvinos, K. Mylona, Tuning Parameter Estimation in Penalized Least Squares Methodology, *Communications in Statistics - Simulation and Computation* 40:9, 1444-1457 (2011).
- [2] P. Craven, G. Wahba, Smoothing noisy data with spline functions, *Numerical Mathematics* 31, 377-403 (1979).
- [3] J. Fan, R. Li, Variable selection via nonconcave penalized likelihood and its oracle properties, *J. Amer. Statist. Assos.* 96, 1348-1360 (2001).
- [4] M. Mitrouli, P. Roupa, Estimates for the generalized cross-validation function via an extrapolation and statistical approach, submitted.

D.

Data Assimilation

Matlab implementation of a spectral algorithm for the seriation problem

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Abstract

Seriation is an important ordering problem which consists of finding the best ordering of a set of units whose interrelationship can be defined by a bipartite graph. It is frequently used in archaeology and it has important applications in many fields such as anthropology, biology, bioinformatics, genetics and psychology. We will present a Matlab implementation of an algorithm for spectral seriation by Atkins et al. [1], based on the use of the Fiedler vector of the Laplacian matrix associated to the problem and which encodes the set of admissible solutions into a particular data structure called PQ-tree.

We will discuss the case of the presence of a multiple Fiedler value which may have a substantial influence on the computation of an approximate solution to the seriation problem and some numerical examples for which is not possible to find an exact solution [2].

References

- [1] Atkins, Jonathan E. and Boman, Erik G. and Hendrickson, Bruce. *A spectral algorithm for seriation and the consecutive ones problem*. SIAM Journal on Computing, **28**(1), 297-310 (1998)
- [2] A. Concas, C. Fenu, and G. Rodriguez. *PQser: a Matlab package for spectral seriation*. arXiv:1711.05677 [cs.MS], 2017.

Study of boundary conditions in the Iterative Filtering method for the decomposition of non-stationary signals

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Abstract

The decomposition of non-stationary signals is a problem of great interest from the theoretical point of view and has important applications in many different fields. For instance, it occurs in the identification of hidden periodicities and trends in time series relative to natural phenomena (like average troposphere temperature) and economic dynamics (like financial indices). Since standard techniques like Fourier or Wavelet Transform are unable to properly capture non-stationary phenomena, in the last years several ad hoc methods have been proposed in the literature. Such techniques provide iterative procedures for decomposing a signal into a finite number of simple components, called Intrinsic Mode Functions (IMFs). At this regard, we recall the well-known and widely used Empirical Mode Decomposition (EMD), an algorithm conceived in 1998 by Huang and his research team at NASA [1]. Since this strategy is empirical and lacks of theoretical foundations, recently the Iterative Filtering (IF) method has been proposed [2]. IF is based on ideas similar to EMD, but unlike EMD allows to make a mathematical analysis of method properties. In this talk we focus on investigating the use of different Boundary Conditions (BCs) in IF, which give rise to different matrix structures. The results presented are based on tools developed in the context of image restoration [3, 4]. Numerical experiments show that a suitable choice of BCs is able to improve in a meaningful way the quality of signal decomposition in IMFs computed by IF method.

References

- [1] N. E. HUANG, Z. SHEN, S. R. LONG, M. C. WU, H. H. SHIH, Q. ZHENG, N.-C. YEN, C. C. TUNG, H. H. LIU, *The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis*, Proceedings of Royal Society of London A **454**, pp. 903–995 (1998).
- [2] L. LIN, Y. WANG AND H. M. ZHOU, *Iterative filtering as an alternative algorithm for empirical mode decomposition*, Advances in Adaptive Data Analysis **1**, pp. 543–560 (2009).
- [3] M. K. NG, R. H. CHAN AND W. C. TANG, *A fast algorithm for deblurring models with Neumann boundary conditions*, SIAM J. Sci. Comput. **21**, pp. 851–866 (1999).
- [4] S. SERRA CAPIZZANO, *A note on antireflective boundary conditions and fast deblurring models*, SIAM J. Sci. Comput. **25**, pp. 1307–1325 (2003).

Parametrizations and prediction

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Abstract

We consider the problem of climate change detection. The years taken into account and the annual mean temperature are denoted by $0, \dots, n$ and t_0, \dots, t_n , respectively. We propose to predict the temperature t_{n+1} using the data t_0, \dots, t_n . We construct a list of parametrizations $(\Theta^{(l)} : l = 0, \dots, n+1)$ of the Euclidean spaces $(\mathbb{R}^{l+1} : l = 0, \dots, n+1)$ adapted to the prediction of t_{n+1} . We analyse how the parametrization affects the prediction and also propose confidence interval prediction without using any probabilistic model. We illustrate our results for the annual mean temperature of France and Morocco.

References

- [1] P. Craven, G. Wahba, Smoothing noisy data with spline functions, Numer. Math. 31 (4) (1978) 377–390.
- [2] C. deBoor, A Practical Guide to Spline, Springer (1978) Springer.
- [3] A. Dermoune, C. Preda, Estimation of noisy cubic spline using a natural basis, Annals of the University of Craiova, Mathematics and Computer Science Series 43 (1) (2016) 33–52.
- [4] A. Dermoune, C. Preda, Parametrizations, fixed and random effects, Journal of Multivariate Analysis 154 (2017) 162–176.

New conformal map for the Sinc approximation for exponentially-decaying functions over the semi-infinite interval

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Abstract

The Sinc approximation is a highly-efficient approximation formula for analytic functions expressed as

$$F(x) \approx \sum_{k=-M}^N F(kh) \operatorname{sinc}(x/h - k), \quad x \in (-\infty, \infty),$$

where $\operatorname{sinc}(x) = \sin(\pi x)/(\pi x)$. This approximation gives exponential convergence if $|F(x)|$ decays exponentially as $x \rightarrow \pm\infty$. Here, we should also note that the target interval to be considered is the infinite interval $(-\infty, \infty)$, and accordingly F should be defined over the infinite interval. If the function to be approximated decays exponentially but is defined over the semi-infinite interval $(0, \infty)$, for example $f(t) = \sqrt{t}e^{-t}$, Stenger [1] proposed to employ a conformal map

$$t = \psi(x) = \operatorname{arcsinh}(e^x),$$

by which the transformed function $f(\psi(x))$ is defined over $(-\infty, \infty)$ and decays exponentially as $x \rightarrow \pm\infty$. However, conformal map performing such a role is not unique; if we employ another conformal map, the convergence rate may be improved. In fact, in the area of numerical integration, improvement of the convergence rate has been reported [2, 3] by replacing the conformal map $t = \psi(x)$ with

$$t = \phi(x) = \log(1 + e^x).$$

Motivated by the fact, this study proposes to combine the Sinc approximation with $t = \phi(x)$ instead of $t = \psi(x)$. A computable error bound for the proposed approximation formula is also given.

References

- [1] F. STENGER: *Numerical Methods Based on Sinc and Analytic Functions*, Springer-Verlag, New York, 1993.
- [2] T. OKAYAMA and K. MACHIDA: Error estimate with explicit constants for the trapezoidal formula combined with Muhammad–Mori’s SE transformation for the semi-infinite interval, *JSIAM Letters*, Vol. 9 (2017), 45–47.
- [3] R. HARA and T. OKAYAMA: Explicit error bound for Muhammad–Mori’s SE-Sinc indefinite integration formula over the semi-infinite interval, *Proceedings of the 2017 International Symposium on Nonlinear Theory and its Applications*, 2017, 677–680.

Perfect matrices of Lagrange differences for the interpretation of dynamics of the cardiovascular system

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Abstract

ECG analysis is the main and the most studied noninvasive technique used for the contemporary investigation of the functionality of the cardiovascular system. ECG parameters are effectively used for the identification of various heart rate and conductivity defects, different heart hypertrophies and ischemic processes. Cardiac time intervals are sensitive markers of cardiac dysfunction, even when it goes unrecognized by conventional echocardiography [1]. Furthermore, interrelations between ECG parameters is still an active area of research.

In this talk we present the concept of perfect matrices of Lagrange differences which are used to investigate the relationships between two ECG parameters RR and JT intervals that are recorded during the bicycle ergometry experiment. The concept of the perfect matrix of Lagrange differences, its parameters, the construction of the load function and the corresponding optimization problem, the introduction of internal and external smoothing, embedding of the scalar parameter time series into the phase plane - all these computational techniques allow visualization of complex dynamical processes taking place in the cardiovascular system during the load and the recovery processes.

The proposed technique allows to observe the "collapse of complexity" at the end of the bicycle stress test, temporary stabilization of transient attractors during the load, rich dynamical behavior of the heart system during the recovery process.

References

- [1] Biering-Srensen T, Mogelvang R, de Knecht MC, Olsen FJ, Galatius S, et al. (2016) Cardiac Time Intervals by Tissue Doppler Imaging M-Mode: Normal Values and Association with Established Echocardiographic and Invasive Measures of Systolic and Diastolic Function. PLOS ONE 11(4): e0153636.

The inverse and variational data assimilation problem on finding the heat flux in the sea thermodynamics model

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Abstract

The methods of data assimilation have become an important tool for analysis of complex physical phenomena in various fields of science and technology. These methods allow us to combine mathematical models, data resulting from observations and a priori information. The problems of variational data assimilation can be formulated as optimal control to find unknown model parameters such as initial and/or boundary conditions, right-hand sides in the model equations (forcing terms), distributed coefficients, based on minimization of the cost function related to observations. A necessary optimality condition reduces an optimal control problem to an optimality system which involves the model equations, the adjoint problem, and input data functions

In this work the variational data assimilation problems in the Baltic Sea water area were formulated and studied [1, 3]. We assume, that the unique function which is obtained by observation data processing is the function of Sea Surface Temperature (SST) and we permit that the function is known only on a part of considering area (for example, on a part of the Baltic Sea). Numerical experiments on restoring the ocean heat flux and obtaining solution of the system (temperature, salinity, velocity, and sea surface level) of the Baltic Sea primitive equation hydrodynamics model [2] with assimilation procedure were carried out with the use of the data error covariance matrix. The spatial resolution of the model grid with respect to the horizontal variables is $0.0625^{\circ} \times 0.03125^{\circ}$ degree. The results of the numerical experiments are presented.

This study was supported by the Russian Foundation for Basic Research (project 16-01-00548) and project 14-11-00609 by the Russian Science Foundation.

References

- [1] Agoshkov V.I., Parmuzin E.I., Zakharova N.B., Zalesny V.B., Shutyaev V.P., Gusev A.V. Variational assimilation of observation data in the mathematical model of the Baltic Sea dynamics, *Russ. J. Numer. Anal. Math. Modelling*, 2015, V. 30, No. 4, pp. 203-212.
- [2] Zalesny V.B., Gusev A.V., Chernobay S.Yu., Aps R., Tamsalu R., Kujala P., Ryttonen J. The Baltic Sea circulation modelling and assessment of marine pollution, *Russ. J. Numer. Analysis and Math. Modelling*, 2014, V 29, No. 2, pp. 129-138.
- [3] Parmuzin E.I., Agoshkov V.I., Zakharova N.B., and Shutyaev V.P. Variational assimilation of mean daily observation data for the problem of sea hydrothermodynamics // *Russ. J. Numer. Anal. Math. Modelling*, 2017, v. 32, no. 3, pp. 187-195.

Sensitivity of the optimal solution of variational data assimilation problems

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Abstract

The problem of variational data assimilation for a nonlinear evolution model is formulated as an optimal control problem to find the unknown parameters of the model. We study the problem of sensitivity of the optimal solution via variational data assimilation with respect to observation errors. On the basis of relations between the error of the optimal solution and the errors of observational data through the Hessian of the cost functional, the algorithms are developed and justified for calculating the coefficients of sensitivity as the norms of the response operators occurring in the equations for errors. A numerical study of the sensitivity of the optimal solution on the example of the problem of variational data assimilation of sea surface temperature to restore the heat flows for the model of thermodynamics is presented. Numerical examples for data assimilation in the Baltic Sea dynamics model are given. This work was carried out within the Russian Science Foundation project 14-11-00609 (numerical experiments) and the project 18-01-00267 of the Russian Foundation for the Basic Research.

References

- [1] Gejadze I., Le Dimet F.-X., Shutyaev V. On analysis error covariances in variational data assimilation. *SIAM J. Sci. Comput.*, 2008, v.30, no.4, pp. 1847–1874.
- [2] Gejadze I., Le Dimet F.-X., Shutyaev V. On optimal solution error covariances in variational data assimilation problems. *J. Comp. Phys.*, 2010, v.229, pp. 2159–2178.
- [3] Gejadze I., Shutyaev V.P., Le Dimet F.-X. Analysis error covariance versus posterior covariance in variational data assimilation. *Q. J. R. Meteorol. Soc.*, 2013, v.139, pp. 1826–1841.
- [4] Gejadze, I.Yu., Shutyaev, V. On gauss-verifiability of optimal solutions in variational data assimilation problems with nonlinear dynamics. *J. Comp. Phys.*, 2015, v.280, pp.439–456.
- [5] Le Dimet, F.-X., Shutyaev, V., Parmuzin, E.I. Sensitivity of functionals with respect to observations in variational data assimilation. *Russ. J. Numer. Anal. Math. Modelling*, 2016, v.31, no.2, pp.81–91.
- [6] Shutyaev, V., Gejadze, I., Vidard, A., and Le Dimet, F.-X. Optimal solution error quantification in variational data assimilation involving imperfect models. *Int. J. Numer. Meth. Fluids*, 2017, v.83, no.3, pp.276–290.
- [7] Gejadze, I., Oubanas, H., and Shutyaev, V. Implicit treatment of model error using inflated observation-error covariance. *Q.J.R. Meteorol. Soc.*, 2017, v.143, pp.2496–2508.

ICS “INM RAS Baltic Sea” for the marine environment state monitoring

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Abstract

Development of Informational Computational Systems (ICS) for data assimilation procedures is one of multidisciplinary problems. To study and solve these problems one needs to apply modern results from different disciplines and recent developments in mathematical modeling, theory of adjoint equations and optimal control, inverse problems, numerical methods theory, numerical algebra, scientific computing and processing of satellite data.

In this work the results on the ICS development are presented. We discuss practical problems studied by ICS “INM RAS Baltic Sea“. The System includes numerical model of the Baltic Sea thermodynamics, the oil spill model describing the propagation of a slick at the Sea surface (Agoshkov, Aseev et al., 2014) and the optimal ship route calculating block (Agoshkov, Zayachkovsky et al., 2014). The ICS is based on the INMOM numerical model of the Baltic Sea thermodynamics (Zalesny et al., 2013). It is possible to calculate main hydrodynamic parameters (temperature, salinity, velocities, sea level) using user-friendly interface of the ICS. The System includes data assimilation procedures (Agoshkov, Parmuzin et al., 2015) and one can use the block of variation assimilation of the sea surface temperature in order to obtain main hydrodynamic parameters. Main possibilities of the ICS and several numerical experiments are presented in the work.

The study was supported by the Russian Science Foundation (project 14-11-00609).

References

- [1] V. B. Zalesny, A. V. Gusev, V. O. Ivchenko, R. Tamsalu, and R. Aps, Numerical model of the Baltic Sea circulation // Russ. J. Numer. Anal. Math. Modelling. 2013. Volume 28, Issue 1, Pages 85-100.
- [2] V.I. Agoshkov, A.O. Zayachkovskiy, R. Aps, P. Kujala, and J. Ryttonen. Risk theory based solution to the problem of optimal vessel route // Russian Journal of Numerical Analysis and Mathematical Modelling. 2014. Volume 29, Issue 2, Pages 69-78.
- [3] Agoshkov, V., Aseev, N., Aps, R., Kujala, P., Ryttonen, J., Zalesny, V. The problem of control of oil pollution risk in the Baltic Sea // Russian Journal of Numerical Analysis and Mathematical Modelling. 2014. Volume 29, Issue 2, Pages 93-105.
- [4] Agoshkov V.I., Parmuzin E.I., Zakharova N.B., Zalesny V.B., Shutyaev V.P., Gusev A.V. Variational assimilation of observation data in the mathematical model of the Baltic Sea dynamics // Russ. J. Numer. Anal. Math. Modelling, 2015, V. 30, No. 4, Pages 203-212.

E.

ODEs, PDEs and Integral Equations

Pseudo-symplectic methods for stochastic Hamiltonian systems

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Abstract

Consider the stochastic autonomous Hamiltonian system in the sense of Stratonovich

$$dP_t^i = -\frac{\partial H_0}{\partial Q_t^i} dt - \sum_{r=1}^d \frac{\partial H_r}{\partial Q_t^i} \circ dw_t^r, \quad dQ_t^i = \frac{\partial H_0}{\partial P_t^i} dt + \sum_{r=1}^d \frac{\partial H_r}{\partial P_t^i} \circ dw_t^r, \quad (1)$$

where $P_0 = p$, $Q_0 = q$, P , Q , p , q are n -dimensional column vectors, and w_t^r , $r = 1, \dots, n$ are independent standard Wiener processes. The flow $\phi_t(p, q) = (P_t(p, q), Q_t(p, q))^T$ of (1) preserves the symplectic structure [3]:

$$\left(\frac{\partial \phi_t}{\partial y_0} \right)^T J \left(\frac{\partial \phi_t}{\partial y_0} \right) = J, \quad J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \quad y_0 = (p, q)^T. \quad (2)$$

Numerical simulations over a long time interval show that symplectic numerical schemes give more accurate approximations for the solutions of stochastic Hamiltonian systems [3]. Several strong and weak symplectic schemes were proposed [3], [1], but unless we consider special stochastic Hamiltonian systems, symplectic schemes are implicit [3]. However, particularly in large Monte Carlo simulations, explicit schemes are desirable in terms of computing time.

For stochastic Hamiltonian systems with additive noise, explicit pseudo-symplectic methods are constructed in [4], and they show good performance for long time computations. In [4] a pseudo-symplectic numerical method $y_1 = \Phi_h(y_0)$, with time step h , of mean-square order (M, N) , $N > M$ is defined as a method of mean square order M that satisfies

$$\left(E \left\| \left(\frac{\partial \Phi_h}{\partial y_0} \right)^T J \left(\frac{\partial \Phi_h}{\partial y_0} \right) - J \right\|^2 \right)^{1/2} = \mathcal{O}(h^{N+1}). \quad (3)$$

We extend the approach used in the deterministic case [2], and we construct explicit pseudo-symplectic schemes in the strong and weak sense for the general stochastic Hamiltonian system (1). We illustrate the properties of these methods thorough several numerical experiments.

References

- [1] C.A. Anton, J. Deng, and Y.S. Wong. Weak symplectic schemes for stochastic Hamiltonian equations. *ETNA*, 43:1–20, 2014.
- [2] A. Aubry and P. Chartier. Pseudo-symplectic Runge-Kutta methods. *BIT Numer. Math.*, 38(3):439–461, 1998.
- [3] G. N. Milstein, Y. M. Repin, and M. V. Tretyakov. Numerical methods for stochastic systems preserving symplectic structure. *SIAM J. Numer. Anal.*, 40(4):1583–1604, 2002.
- [4] X. Niu, J. Cui, J. Hong, and Z. Liu. Explicit pseudo-symplectic methods for stochastic Hamiltonian systems. *BIT Numer. Math.*, 2017. <https://doi.org/10.1007/s10543-017-0668-7>.

Recovering the electrical conductivity of the soil via linear integral equations

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Abstract

This work investigates a linear model that involves integral equations of the first kind defined on the positive semiaxes used to describe the interaction of an electromagnetic field with the soil [3]. The aim is to detect, by non destructive investigation of soil properties, inhomogeneities in the ground as well as the presence of particular conductive substances.

To find the solution of the problem, we propose different numerical methods based on splines, Bernstein polynomials or Laguerre orthogonal polynomials combined with some suitable regularization techniques as the Truncated Singular Value Decomposition and Tikhonov regularization [1, 2].

Finally, we compare the results obtained by each method mentioned above on synthetic data sets.

References

- [1] P. C. Hansen. *Rank-Deficient and Discrete Ill-Posed Problems*, SIAM, Philadelphia, 1998.
- [2] R. Kress. *Linear Integral Equation*, Springer, 1999.
- [3] J. D. McNeill. *Electromagnetic terrain conductivity measurement at low induction numbers*, Technical Report TN-6 Geonics Limited, 1980.

On a Collocation-quadrature Method for the Singular Integral Equation of the Notched Half-plane Problem

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Abstract

This contribution deals with a notched half plane problem of two-dimensional elasticity theory, which considers a straight crack of normalized length 2 perpendicular to and ending at the boundary of the elastic half plane. The problem can be modelled by a hypersingular integral equation, the solution of which is the crack opening displacement. For the numerical solution of this equation we propose polynomial collocation-quadrature methods, which look for an approximation of the derivative of the crack opening displacement. This derivative is the solution of a Cauchy singular integral equation with additional fixed singularities, which is given by the equation

$$\frac{1}{\pi} \int_{-1}^1 \left[\frac{1}{y-x} + \mathbf{h} \left(\frac{1+x}{1+y} \right) \frac{1}{1+y} \right] v'(y) dy = f(x), \quad x \in (-1, 1),$$

where the right hand side $f : [-1, 1] \rightarrow \mathbb{C}$ is smooth and where

$$\mathbf{h}(t) = -\frac{1}{1+t} + \frac{6t}{(1+t)^2} - \frac{4t^2}{(1+t)^3}.$$

By using C*-algebra techniques as well as results from a previous work [1], we present necessary and sufficient conditions for the stability of the collocation-quadrature methods as well as numerical results. In contrast to former works (cf. [1] and [2]) our results are based on a new approach, which takes into account the “natural” asymptotic of the solution at the endpoints of the integration interval and for which until now no criterion for stability is known.

References

- [1] P. Junghanns, R. Kaiser, and G. Mastroianni. Collocation for singular integral equations with fixed singularities of particular Mellin type. *Electron. Trans. Numer. Anal.*, 41:190–248, 2014.
- [2] P. Junghanns, R. Kaiser, and D. Potts. Collocation–quadrature methods and fast summation for Cauchy singular integral equations with fixed singularities. *Linear Algebra Appl.*, 491:187–238, 2016.
- [3] P. Junghanns and R. Kaiser. On a collocation-quadrature method for the singular integral equation of the notched half-plane problem, *Operator Theory: Advances and Applications*, 259:413–462, 2017.

Subspace methods for three-parameter eigenvalue problems

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Abstract

In many applications a PDE has to be solved on a domain that allows the use of the method of separation of variables. In several coordinate systems this leads to two- or three-parameter eigenvalue problems (2EP or 3EP), an example is the Helmholtz equation in ellipsoidal and paraboloidal coordinates. While there exist several subspace methods for 2EP, extensions to 3EP are not straightforward. We propose two subspace methods for 3EP, a subspace iteration with Arnoldi expansion and a Jacobi–Davidson type method, which we generalize from their 2-parameter counterpart and add important new features. Methods are implemented in the Matlab toolbox MultiParEig [1].

In the generic case, separation of variables applied to a separable boundary value problem, followed by a discretization, leads to a multiparameter eigenvalue problem of the form

$$A_{i0} x_i = \sum_{j=1}^k \lambda_j A_{ij} x_i, \quad i = 1, \dots, k, \quad (1)$$

where $k \in \{2, 3\}$ and $A_{ij} \in \mathbb{C}^{n_i \times n_i}$ for $i = 1, \dots, k$ and $j = 0, \dots, k$. A k -tuple $(\lambda_1, \dots, \lambda_k)$ is an eigenvalue if it satisfies (1) for nonzero vectors x_1, \dots, x_k and the corresponding eigenvector is $x_1 \otimes \dots \otimes x_k$. By introducing the so-called $k \times k$ operator determinants

$$\Delta_0 = \begin{vmatrix} A_{11} & \cdots & A_{1k} \\ \vdots & & \vdots \\ A_{k1} & \cdots & A_{kk} \end{vmatrix}_{\otimes} \quad \text{and} \quad \Delta_i = \begin{vmatrix} A_{11} & \cdots & A_{1,i-1} & A_{10} & A_{1,i+1} & \cdots & A_{1k} \\ \vdots & & \vdots & \vdots & & & \vdots \\ A_{k1} & \cdots & A_{k,i-1} & A_{k0} & A_{k,i+1} & \cdots & A_{kk} \end{vmatrix}_{\otimes}$$

for $i = 1, \dots, k$, where the Kronecker product \otimes is used instead of multiplication, we get matrices $\Delta_0, \dots, \Delta_k$ of order $n_1 \cdots n_k$. If Δ_0 is nonsingular, then $\Delta_0^{-1} \Delta_1, \dots, \Delta_0^{-1} \Delta_k$ commute and (1) is equivalent to a system of generalized eigenvalue problems $\Delta_i z = \lambda_i \Delta_0 z$, $i = 1, \dots, k$, where $z = x_1 \otimes \dots \otimes x_k$. This relation enables us to compute all eigenvalues of (1) if the Δ -matrices are small. However, usually even for $k = 2$ the Δ -matrices are so large that it is not efficient or even not feasible to compute all the eigenvalues.

In many applications we need only the smallest eigenvalues of $\Delta_k z = \mu \Delta_0 z$. For $k = 2$ we can efficiently apply the Krylov subspace methods since we can exploit the structure of Δ_2 to reduce the complexity of solving a system with Δ_2 from $\mathcal{O}(n_1^3 n_2^3)$ to $\mathcal{O}(n_1^3 + n_2^3)$. For $k = 3$ it remains open how to reduce the complexity of solving a system with Δ_3 below $\mathcal{O}(n_1^3 n_2^3 n_3^3)$. The new methods for 3EP overcome this obstacle and we will show how they can be used to compute accurate eigenfunctions for the ellipsoidal wave equations and Baer wave equations efficiently.

References

- [1] B. Plestenjak, MultiParEig, www.mathworks.com/matlabcentral/fileexchange/47844-multipareig

F.
Control Theory

Linear control systems partially ordered by the sharp partial order

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Abstract

Matrix partial orders are a useful tool in several research areas. In [1] the authors show that, for two matrices A and B of the same size, $A \leq B$ implies that B can be seen as a perturbation of A for the most common matrix partial orders. We apply this theory in control linear systems [2]. Specifically, we compare two control linear systems whose state matrices are related under the sharp partial order. It should be interesting to have a relationship between the solutions of both systems. In this work we present this relation by finding that the difference between both solutions is given by the perturbation between the involved state matrices. Moreover, an upper bound for this difference is found.

Acknowledgements

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References

- [1] S.K. Mitra, P. Bhimasankaram, S.B. Malik. *Matrix Partial Orders, Shorted Operators and Applications*. World Scientific Publishing Company, London, 2010.
- [2] T. Kaczorek *Linear Control Systems, Vol 1 and Vol 2*. John Wiley and Sons Inc., New York, 1992.

Distance to \mathcal{R} -Uncontrollability

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Abstract

We consider the continuous time descriptor system $E\dot{x}(t) = Ax(t) + Bu(t)$, denoted by (E, A, B) , with $E \in \mathbb{R}^{n \times n}$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $t \geq 0$ representing time. A fundamental concept associated with descriptor systems is that of controllability. Intuitively, this is the ability of x to move from an initial to a final value in finite time by some control action $u(t)$. If E is allowed to be singular, x is restricted to the so-called *reachable* subspace $\mathcal{R} \subseteq \mathbb{R}^n$. This gives rise to the concept of controllability within \mathcal{R} , termed \mathcal{R} -controllability, and to the concept of the *distance* of (E, A, B) from the nearest \mathcal{R} -uncontrollable system, also termed the *radius* of (E, A, B) .

In this work we concentrate on a special kind of a descriptor system termed the *semiexplicit* system (J, A, B) , where $J = \text{diag}(I, 0)$. Often, descriptor systems appear naturally in this form, but also any system (E, A, B) is equivalent to some semiexplicit system via an equivalence relation $(J, A, B) \equiv \text{diag}(\Sigma^{-1}, I) Q^T (E, A, B) \text{diag}(P, P, I)$, using for example the singular value decomposition $Q^T E P = \text{diag}(\Sigma, 0)$ of E . The *radius* μ of (J, A, B) , can be defined, and be shown equal to:

$$\begin{aligned} \mu &\equiv \min_{(\delta A, \delta B) \in \mathbb{C}^{n \times (n+m)}} \|(\delta A, \delta B)\|_2 : (J, A + \delta A, B + \delta B) \text{ is } \mathcal{R}\text{-uncontrollable} \\ &= \min_{\lambda \in \mathbb{C}} \underbrace{\sigma_{\min}(B, A - \lambda J)}_{\sigma(\lambda)}, \text{ where } \sigma_{\min} \text{ denotes the smallest singular value.} \end{aligned}$$

The contribution of this research is the efficient estimation of μ without resorting to algorithms that estimate the radius of a general descriptor system since they all consider, as they should, perturbations on E and consequently give rise to algorithms that are unnecessarily complicated for semiexplicit systems. Furthermore some of them do not allow E to be singular. On the other hand, with the exception of one paper, which will be discussed, none of the papers that estimate the radius of (I, A, B) can be altered, at least in a straightforward way, in order to handle (J, A, B) , where J is allowed to be singular.

The study of $\sigma(\lambda)$ shows that it possesses several local minima clustered in a specified area. To address this, we developed a novel algorithm for the computation of the global minimum of $\sigma(\lambda)$, which we termed the *feedback search algorithm*. We have implemented this algorithm in MATLAB and applied it successfully on $\sigma(\lambda)$ and on two other related functions using several random examples as well as examples from the literature.

G.
Linear Algebra

Sharp bounds for eigenvalues of the generalized k, m -step Fibonacci matrices

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Abstract

For two given integers $k = 1, 2, \dots, m = 0, 1, \dots$, and the nonnegative real constants c_1, c_2, \dots, c_k , the n -th term, f_n of the *generalized k, m -step Fibonacci sequence* is given by the recursive formula

$$\begin{aligned} f_n &= c_1 f_{n-m-1} + c_2 f_{n-m-2} + \dots + c_k f_{n-m-k} \\ &= \sum_{j=m+1}^{k+m} c_{j-m} f_{n-j}, \quad \text{for every } n \geq k + m + 1, \end{aligned} \quad (1)$$

with

$$f_1 = f_2 = \dots = f_{k+m} = 1.$$

Using (1) the generalized k, m -step Fibonacci sequence can be represented by the *generalized k, m -Fibonacci matrices*, which are defined in [1] and some bounds for the spectral radius of the matrices are discussed. In this paper, the powers of the generalized k, m -step Fibonacci matrices are investigated and closed formulas for their entries are derived, related to the suitable terms of the k, m -step Fibonacci sequences in order to develop the properties of the irreducibility and primitivity of the Fibonacci matrices. New upper and lower bounds for the spectral radius and the modulus of the remaining eigenvalues of the generalized k, m -Fibonacci matrices are presented. Some known results for the nonnegative matrices in the literature are generalized and improved, [2, 3, 4, 5]. Applications of the results are given in the tridiagonal k -Toeplitz matrices.

References

- [1] M. Adam, N. Assimakis and G. Tzialis, Generalized k, m -step Fibonacci sequences and matrices, Proceedings of the 12th International Conference of AHA-Algebraic Hyperstructures and its Applications, *Int. J. of Algebraic Hyperstructures and Its Applications*, 2(1), (2015), 125-134. http://www.ijaha.org/article_42063_34a5cd1c82a0aa8fcb4116ca147130df.pdf
- [2] A. Brauer and I.C. Gentry, Bounds for the greatest characteristic root of an irreducible nonnegative matrix, *Linear Algebra Appl.*, 8, (1974), 105-107.
- [3] A. Melman, Upper and lower bounds for the Perron root of a nonnegative matrix, *Linear and Multilinear Algebra*, 61(2), (2013), 171-181.
- [4] M. El-Mikkawy and T. Sogabe, A new family of k -Fibonacci numbers, *Applied Mathematics and Computation*, 215, (2010), 4456-4461.
- [5] R. Xing and B. Zhou, Sharp bounds for the spectral radius of nonnegative matrices, *Linear Algebra Appl.*, 449, (2014), 194-209.

Further insights into the embedding properties of Hadamard matrices

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Abstract

Hadamard matrices have many applications in several mathematical areas due to their special form and the numerous properties that characterize them [2, 4]. Based on a recently developed relation between minors of Hadamard matrices [5] and using tools from calculus and elementary number theory, the present work highlights a direct way to investigate the conditions under which an Hadamard matrix of order $n - k$ can or cannot be embedded in an Hadamard matrix of order n . In this study, we analyzed the embedding properties of Hadamard matrices via their minors and revisited the method of proving $H_{n-k} \notin H_n$ when $k < \frac{n}{2}$, which was originally presented in [1]. A systematic approach was followed to this problem, first by looking at the cases $H_{n-4} \notin H_n$ and $H_{n-8} \notin H_n$, and then, considering the general case $H_{n-k} \notin H_n$. The results obtained allowed us to study the problem further when $k \geq \frac{n}{2}$, which may reveal a characteristic embedding pattern for all Hadamard matrices. In particular, for $k = \frac{n}{2}$ it is known that $H_k \in H_{2k}$ and for $k > \frac{n}{2}$ we infer that a Hadamard matrix of order $n - k$ may exist embedded in a Hadamard matrix of order n , that is $H_{n-k} \in H_n$, if the Hadamard matrix of order n has a $k \times k$ submatrix with minor $p 2^{k-1}$ and the value of p is specifically given by the function:

$$p := \mathcal{P}(n, k) = 2 \left(\frac{n}{4} \right)^{\frac{k}{2}} \left(\frac{n-k}{n} \right)^{\frac{n-k}{2}} \text{ for } \frac{n}{2} \leq k < n \text{ and } \begin{cases} n = 8, 12, 16, \dots \\ k = 4, 8, 12, \dots \end{cases}$$

The results obtained also provide answers to the problem of determining the values of the spectrum of the determinant function [3] for specific orders of minors of Hadamard matrices.

References

- [1] J.H.E. Cohn, *Hadamard matrices and some generalisations*, The American Mathematical Monthly, 72.5, (1965), 515-518.
- [2] K.J. Horadam, *Hadamard Matrices and Their Applications*, Princeton University Press, Princeton, NJ, 2007.
- [3] W.P. Orrick and B. Solomon, *Spectrum of the determinant function*, <http://www.indiana.edu/~maxdet/spectrum.html>
- [4] J. Seberry and M. Mitrouli, *Some remarks on Hadamard matrices*, Cryptogr. Commun., 2, (2010), 293-306.
- [5] F. Szollosi, *Exotic Complex Hadamard matrices and their equivalence*, Cryptogr. Commun., 2, (2010), 187-198.

Generalized SR calculus for derivatives of split-quaternionic functions

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Abstract

This paper is to give the relation between the derivatives of split-quaternion valued functions in \mathbf{S} and the corresponding functions of four real variables in \mathbb{R}^4 , in order to obtain a quaternion extension of the **HR** calculus termed the **SR** calculus. Due to non-commutativity of split-quaternion product, we induces the **SR** calculus as the left- and right-hand versions of derivatives of split-quaternionic functions. In particular, we show that for real functions of split-quaternion variables the left and right **SR** derivatives are identical. That is, the use of the left/right **SR** derivative does not affect to practical applications of (split-)quaternion optimization. Finally, we consider several fundamental theorems in (split-)quaternion calculus, based on the **SR** derivatives to enable expansion to (split-)quaternions in practical applications.

References

- [1] Xu, D., Jahanchahi, C., Took, C. C., Mandic, D. P. (2015). Enabling quaternion derivatives: the generalized HR calculus. *Royal Society open science*, 2(8), pp.150–255.
- [2] Mandic, D. P., Jahanchahi, C., Took, C. C. (2011). A quaternion gradient operator and its applications. *IEEE Signal Processing Letters*, 18(1), pp.47–50.
- [3] Jahanchahi, C., Took, C. C., Mandic, D. P. (2012, March). On gradient calculation in quaternion adaptive filtering. In *Acoustics, Speech and Signal Processing (ICASSP), 2012 IEEE International Conference*.
- [4] Ell, T. A., & Sangwine, S. J. (2007). Quaternion involutions and anti-involutions. *Computers & Mathematics with Applications*, 53(1), 137-143.
- [5] Sudbery, A. (1979, March). Quaternionic analysis. In *Mathematical Proceedings of the Cambridge Philosophical Society* (Vol. 85, No. 2, pp. 199-225). Cambridge University Press.
- [6] Jahanchahi, C., Took, C. C., & Mandic, D. P. (2010, July). On HR calculus, quaternion valued stochastic gradient, and adaptive three dimensional wind forecasting. In *Neural Networks (IJCNN), The 2010 International Joint Conference on* (pp. 1-5). IEEE.
- [7] Kreutz-Delgado, K. (2009). The complex gradient operator and the CR-calculus. *arXiv preprint arXiv:0906.4835*.

Accurate and Efficient Traces of Beta–Wishart Matrices

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Abstract

The eigenvalues of a Wishart random matrix and its trace have long been used in multivariate statistical analysis for a variety of analyses and applications [4]. This class was recently generalized to any $\beta > 0$ to obtain the class of Beta–Wishart matrices of which the classical real, complex, and quaternion cases correspond to $\beta = 1, 2$, and 4 , respectively [1, 2].

The only known expressions for the eigenvalues and the trace, however, are in terms of infinite series of Jack functions, and in particular, the hypergeometric function of a matrix argument. These series are notoriously slow to converge and have been a computational challenge for decades despite recent progress [3]. The main issue is the exponential number of terms in (a finite truncation of) the expansion of hypergeometric function as a series of Jack functions.

We will present new expressions and a new algorithm for computing the density and distribution of the trace of a Beta–Wishart matrix which is linear in both the size of the matrix *and* the degree of the truncation. This complexity is optimal. Additionally, our new algorithm is subtraction-free, which means that the results will be computed to high relative accuracy in the presence of roundoff errors in floating point arithmetic.

Additionally, we will present new results that allow the computation of the density of the largest eigenvalue of a Beta–Wishart matrix order of magnitude faster than previous results.

References

- [1] Alexander Dubbs, Alan Edelman, Plamen Koev, and Praveen Venkataramana, *The Beta–Wishart ensemble*, J. Math. Phys. **54** (2013), 083507.
- [2] Peter Forrester, *Probability densities and distributions for spiked and general variance Wishart β -ensembles*, Random Matrices: Theory and Applications **2** (2013), no. 4, 1350011.
- [3] Plamen Koev and Alan Edelman, *The efficient evaluation of the hypergeometric function of a matrix argument*, Math. Comp. **75** (2006), no. 254, 833–846.
- [4] R. J. Muirhead, *Aspects of multivariate statistical theory*, John Wiley & Sons Inc., New York, 1982.

On the inverse problem associated to $KA^sK = A^{s+1}$

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Abstract

In previous papers, the authors introduced and characterized a kind of matrices called $\{K, s + 1\}$ -potent [1, 2]. Also, they established a method to calculate these matrices. Some related class of matrices were studied in [3, 4]. The purpose of the present paper is to solve the associated inverse problem. Several algorithms are developed in order to find all involutory matrices K satisfying $KA^{s+1}K = A$ for a given matrix $A \in \mathbb{C}^{n \times n}$ and a given natural number s . The cases $s = 0$ and $s \geq 1$ are separately studied since they produce different situations. In addition, some examples are presented showing the numerical performance of the methods.

Acknowledgements

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References

- [1] L. Lebtahi, O. Romero, N. Thome, Characterizations of $\{K, s + 1\}$ -Potent Matrices and Applications, *Linear Algebra and its Applications*, 436, 293–306, (2012).
- [2] L. Lebtahi, O. Romero, N. Thome, Algorithms for $\{K, s + 1\}$ -potent matrix constructions, *Journal of Computational and Applied Mathematics*, 249, 157–162, (2013).
- [3] J.L. Stuart, Infation matrices and ZME-Matrices that commute with a permutation matrix, *SIAM J. Matrix Analysis Applications*, 9, (3), 408–418, (1998).
- [4] W.F. Trench, Characterization and properties of matrices with k - involutory symmetries II, *Linear Algebra and its Applications*, 432, 2782–2797, (2010).

On the positive definite solutions of the nonlinear matrix equations $X^p = A \pm M^T(X^{-1}\#B)^{-1}M$

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Abstract

The two nonlinear matrix equations $X^p = A + M^T(X^{-1}\#B)^{-1}M$ and $X^p = A \pm M^T(X^{-1}\#B)^{-1}M$ are studied, where $p \geq 1$ is a positive integer, M is an $n \times n$ nonsingular matrix, A is a positive semidefinite matrix and B is a positive definite matrix. We call $C\#D$ the geometric mean of positive definite matrices C and D . We show the existence and uniqueness of the nonlinear matrix equations. Estimates of the positive definite solution are given. Iteration method for finding the numerical solution is proposed.

References

- [1] T. Ando, On the arithmetic-geometric-harmonic mean inequalities for positive definite matrices, *Linear Algebra Appl.* 52-53 (1983) 31-37.
- [2] T. Ando, *Topics on Operator Inequalities*, Lecture Notes Hokkaido University, Sapporo, 1978.
- [3] C. Jung, H.-M. Kim, Y. Lim, On the solution of the nonlinear matrix equation $X^n = f(X)$, *Linear Algebra Appl.* 430 (2009) 2042-2052.
- [4] J.D. Lawson, Y. Lim, The geometric mean, matrices, metrics, and more, *Am. Math. Monthly* 108 (2001) 797-812.

Gershgorin type sets for polynomial eigenvalue problems

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Abstract

New localization results for polynomial eigenvalue problems are obtained, by extending the notions of the Gershgorin set, the generalized Gershgorin set (known also as the \mathcal{A} -Ostrowski set), the Brauer set, and the Dashnic-Zusmanovich set, to the case of matrix polynomials. For each eigenvalues' inclusion set, basic topological and geometrical properties are presented, and illustrative examples are given.

References

- [1] D. Bindel and A. Hood, Localization theorems for nonlinear eigenvalue problems, *SIAM Journal on Matrix Analysis and Applications*, **34**(6) (2013), 1728–1749.
- [2] A. Brauer, Limits for the characteristic roots of a matrix II, *Duke Mathematical Journal*, **14** (1947), 21–26.
- [3] L.S. Dashnic and M.S. Zusmanovich, K voprosu o lokalizacii harakteristicheskikh chisel matricy, *Zhurnal Vychislitel'noi Matematiki i Matematicheskoi Fiziki*, **10**(6) (1970), 1321–1327.
- [4] S. Gershgorin, Über die abgrenzung der eigenwerte einer matrix, *Izvestiya Akademii Nauk SSSR, Seriya Fizicheskaya-Matematicheskaya*, **6** (1931), 749–754.
- [5] I. Gohberg, P. Lancaster and L. Rodman, *Matrix Polynomials*, Society for Industrial and Applied Mathematics, Philadelphia, 2009.
- [6] N.J. Higham and F. Tisseur, Bounds for eigenvalues of matrix polynomials, *Linear Algebra and its Applications*, **358** (2003) 5–22.
- [7] P. Lancaster, *Lambda-Matrices and Vibrating Systems*, Dover Publications, 2002.
- [8] P. Lancaster and M. Tismenetsky, *The Theory of Matrices*, 2nd edition, Academic Press, Orlando, 1985.
- [9] A.S. Markus, *Introduction to the Spectral Theory of Polynomial Operator Pencils*, Translations of Mathematical Monographs, Vol. 71, American Mathematical Society, Providence, 1988.
- [10] A.M. Ostrowski, Über das Nichtverschwinden einer Klasse von Determinanten und die Lokalisierung der charakteristischen Wurzeln von Matrizen, *Compositio Mathematica*, **9** (1951) 209–226.
- [11] R.S. Varga, *Geršgorin and His Circles*, Springer Series in Computational Mathematics, Vol. 36, Springer-Verlag, Berlin, 2004.

H.

Inverse Problems

An inverse scattering problem for an electromagnetic layered ellipsoid

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Abstract

In this work the scattering problem of time-harmonic electromagnetic waves from a two-layered obstacle consisting of a triaxial dielectric ellipsoid with a confocal perfectly conducting ellipsoidal core is considered. A low-frequency formulation of the direct scattering problem is described. Based on near-field or far-field data, a measurement matrix is constructed whose eigenvalues and eigenvectors contain information for the size and the orientation of the ellipsoid. The geometrical method described can be applied for solving inverse electromagnetic scattering problems for spheroids, spheres, needle and discs, considering them as geometrically degenerate forms of the ellipsoid for suitable values of the physical and geometrical parameters.

Domain decomposition method in the problems of oceans and seas hydrothermodynamics based on theory of inverse problems

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Abstract

There are a lot of approaches to the formulation of domain decomposition method. However, some problems connected with the theory and application of domain decomposition method are to be mentioned. The majority of known approaches uses symmetric operators. It may provide simplification of domain decomposition method but it is not acceptable for oceans and seas hydrothermodynamics.

New methodology for constructing the domain decomposition algorithms is based on the theory of optimal control, the results of the theory of inverse and ill-posed problems, the application of adjoint equations and modern iterative processes. Domain is divided into subdomains. To solve subproblems in each subdomain interface conditions are to be set. Some of them become “controls” and are to be found with the solution in subdomains. The second part of interface conditions is written as additional equation to solve the system in terms of least squares. Thus optimal control problem is obtained and it could be solved with the application of known methods. It should be noted that this methodology is applicable to problems with operators of different types, orders and with a different number of independent variables. The work is based on [1].

Domain decomposition method for the Baltic Sea model is numerically studied. The numerical experiments with using and without domain decomposition algorithm are presented and discussed.

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References

- [1] V.I. Agoshkov. Domain Decomposition Methods in Problems of Oceans and Seas Hydrothermodynamics, Institute of Numerical Mathematics RAS, Moscow, 2017 (in Russian).

Truncated SVD methods in the inverse source problems for the advection-diffusion-reaction models with image-type measurement data

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Abstract

The inverse source problems for nonlinear advection-diffusion-reaction models with image-type measurement data are considered. These inverse problems arise in the air quality studies when the measurement data is obtained in the form of time series, vertical concentration profiles or satellite images of concentration fields. The data is of large amount and of different value with respect to the considered inverse problem. The sensitivity operators constructed from the set of the adjoint problem solutions allow us to transform the inverse problem stated as the system of nonlinear ODE or PDE to the family of nonlinear operator equations depending on the given set of orthogonal functions in the space of the measurement results [1]. By the choice of the orthogonal functions the dimensionality of the problem can be reduced thus allowing for the efficient solution of the resulting operator equation with the relevant methods for nonlinear ill-posed operator equations. We consider Newton-Kantorovich type methods based on the truncated SVD. The operator form of the inverse problem can be exploited for the analysis and comparison of the different inverse problem statements, e.g. with the help of the spectral methods. For numerical solution, the discrete-analytical schemes for transport and transformation processes are applied. The schemes are constructed with the use of the locally-adjoint problems [2]. Multidimensional problems are treated according to the splitting technique with respect to spatial dimensions and physical processes. The accuracy of the sensitivity operator calculation is assured by the consistency of the numerical schemes for the direct and adjoint problems in the sense of Lagrange-type identities [3]. The numerical methods are tested on the atmospheric chemistry models.

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References

- [1] Penenko A. V., Nikolaev S. V., Golushko, S. K., Romashenko, A. V., Kirilova I. A. Numerical Algorithms for Diffusion Coefficient Identification in Problems of Tissue Engineering // *Math. Biol. Bioinf.* 2016. Vol. 11. P. 426-444.
- [2] Penenko V., Tsvetova E., Penenko A. Variational approach and Euler's integrating factors for environmental studies // *Computers & Mathematics with Applications*. 2014. Vol. 67. P. 2240-2256.
- [3] Penenko A., Penenko V., Tsvetova E., Grishina A., Antokhin P. Sequential Variational Data Assimilation Algorithms at the Splitting Stages of a Numerical Atmospheric Chemistry Model // *Large-Scale Scientific Computing*, Springer International Publishing. 2018. P. 536-543.

A near-field inverse scattering problem for a thermoelastic ellipsoid

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Abstract

In this work the scattering problem of time-harmonic thermoelastic waves from an ellipsoidal obstacle is considered. The direct scattering problem using low-frequency approximation is formulated. We study a new method using near-field data for solving the corresponding inverse scattering problem determining the size and the orientation of the ellipsoid. A finite number of measurements of the leading order terms of the scattered field in the low-frequency approximation leads to specify the semi-axes of the ellipsoid. The orientation of the ellipsoid is obtained by using a rotation matrix whose elements are in terms of the Euler angles. Corresponding results for geometrically degenerate cases of the ellipsoid such as spheroids, spheres, needles and discs are obtained for appropriate values of the physical and geometrical parameters.

I. Optimization

Stochastic global optimization using tangent minorants for Lipschitz functions

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Abstract

This paper is divided into two parts. In the first one, we elaborate a stochastic algorithm based on the brunch and bound method to minimize objective functions expressed by the expextation of a partially lipschitzian function. The second part deals with the optimization of a semi lower-continuous function. We show how to transform the objective function in order to obtain a situation identical to the one of the first section.

References

- [1] Blomvall. J, Shapiro. A. (2006). Solving multistage asset investment problems by the sample average approximation method. *Mathematical programming*, 108(2-3), 571-595.
- [2] Kleywegt. A.J, Shapiro. A, Homem-de-Mello. T. (2002). The sample average approximation method for stochastic discrete optimization. *SIAM Journal on Optimization*, 12(2), 479-502.
- [3] Norkin. V. (1998). Global optimization of probabilities by the stochastic branch and bound method. In *Stochastic Programming Methods and Technical Applications* (pp. 186-201). Springer Berlin Heidelberg.
- [4] Norkin. V.I, Pflug. G, Ruszczynski.A. A branch and bound method for stochastic global optimization. *Mathematical Programming* 83 (1998)425-450
- [5] Norkin. V.I, Ermoliev. Y.M, Ruszczynski. A. (1998). On optimal allocation of indivisibles under uncertainty. *Operations Research*, 46(3), 381-395.
- [6] Norkin. V.I, Onishchenko. B.O. (2005). Minorant methods of stochastic global optimization. *Cybernetics and Systems Analysis*, 41(2), 203-214.
- [7] Norkin V.I, Onishchenko B.O. A branch and approximate bounds method *Computer mathematics*. Vol. 1. Kiev: Glushkov Ins-t of Cybernetics NAS of Ukraine, 2005. P. 152-160.
- [8] Pagnoncelli. B.K, Ahmed. S, Shapiro. A. (2009). Sample average approximation method for chance constrained programming: theory and applications. *Journal of optimization theory and applications*, 142(2), 399-416.
- [9] Shapiro. A. (2003). Monte Carlo sampling methods. *Handbooks in operations research and management science*, 10, 353-425.
- [10] Shapiro. A, Wardi. Y. (1996). Convergence analysis of stochastic algorithms. *Mathematics of Operations Research*, 21(3), 615-628.
- [11] Shapiro. A, Kleywegt. A. (2002). Minimax analysis of stochastic problems. *Optimization Methods and Software*, 17(3), 523-542.
- [12] Verweij. B, Ahmed. S, Kleywegt. A.J, Nemhauser. G, and Shapiro. A. (2003). The sample average approximation method applied to stochastic routing problems: a computational study. *Computational Optimization and Applications*, 24(2),289-333.
- [13] Yahyaoui. A, Ammar, H. (2017). Global Optimization of Multivariate Holderian Functions Using Overestimators. *Open Access Library Journal*, 4(05), 1.

Constrained Weighted Feature Selection

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Abstract

The aim of feature selection is to identify the most informative and relevant features for a compact and accurate data representation. Generally speaking, feature selection algorithms were handled in supervised and unsupervised learning contexts. However, the semi-supervised context is more realistic where we might have only few labeled data and many others unlabeled. In this regard, another form of supervision information is available, it is based on simple pairwise comparisons [1, 2] and can be more easily obtained compared to class labels. For instance, a data pair is said to be a "must-link constraint" if its data points are similar and a "cannot-link constraint" otherwise. Recently, there was a big interest in constrained clustering that handled choosing the constraints actively and systematically, however, only few worked similarly for feature selection. Unexpectedly, [4] stated that randomly chosen constraint sets can degrade the learning performance.

Therefore, we first suggested a margin-based algorithm, Relief-Sc, for weighting features according to their data discrimination ability. It is said to find a unique relevant feature subset in a closed-form. For that, we modified ReliefF algorithm to adapt the use of cannot-link constraints with the margin concept used in [3]. We also propose to use the systematic way used by [5] to find the points that our feature selection approach might be most uncertain about, and then actively query for constraints upon these particular points. Since we can only query the oracle or expert for few constraints, we finally suggest to extend our algorithm to make use of the unlabeled data together with the chosen set of constraints. In order to validate our proposed algorithm, experiments are achieved on multiple UCI machine learning datasets and the results are prominent.

References

- [1] D.Zhang, S.Chen, and Z.-H.Zhou, "Constraint Score: A new filter method for feature selection with pairwise constraints", *Pattern Recognition*, vol. 41, no. 5, pp. 1440–1451, 2008.
- [2] M. Kalakech, P. Biela, L. Macaire, and D. Hamad, "Constraint scores for semi-supervised feature selection: A comparative study", *Pattern Recognition Letters*, vol. 32, no. 5, pp. 656–665, 2011.
- [3] M. Yang, and J. Song, "A novel hypothesis-margin based approach for feature selection with side pairwise constraints", *Neurocomputing*, vol. 73, no. 16, pp. 2859–2872, 2010.
- [4] I. Davidson, K.L. Wagstaff, and S. Basu, "Measuring constraint-set utility for partitional clustering algorithms", *European Conference on Principles of Data Mining and Knowledge Discovery*, Springer, pp. 115–126, 2006.
- [5] F.L. Wauthier, N. Jojic, and M.I. Jordan, "Active spectral clustering via iterative uncertainty reduction", *Proceedings of the 18th ACM SIGKDD international conference on Knowledge discovery and data mining*, ACM, pp. 1339–1347, 2012.

J.
Appoximation Theory

Rational Approximation for the Inverse of a ϕ -Function of Quasiseparable Matrices

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Abstract

Rational approximation in the form of a partial fraction expansion is especially well-suited to the computation of functions of structured matrices, when such matrices can be inverted cheaply. Here we focus on the inverse of the ϕ_1 function

$$\psi_1(z) = \phi_1(z)^{-1} = \frac{z}{e^z - 1}$$

which is involved in the solution of certain linear differential equations.

We introduce a family of mixed polynomial-rational approximations of $\psi_1(z)$ with the goal of

- computing solutions of the differential problems mentioned above, in the form $\psi_1(A)g$, where A is a quasiseparable matrix and g a vector,
- reconstructing the matrix $\psi_1(A)$.

Numerical experiments illustrate the behavior and the benefits of such an approach.

References

- [1] P. Boito, Y. Eidelman, L. Gemignani, *Efficient Solution of Parameter Dependent Quasiseparable Systems and Computation of Meromorphic Matrix Functions*, Numerical Linear Algebra and Appl., first published 18 Jan 2018, DOI: 10.1002/nla.2141
- [2] P. Boito, Y. Eidelman, L. Gemignani, *Computing the Inverse of a ϕ -Function by Rational Approximation*, arXiv: 1801.04573 [math.NA].

Filling holes with edge reconstruction

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Abstract

Let D be a domain and $H \subset D$. Let $D_1, D_2 \subset D$ be such that $D - H = D_1 \cup D_2$ and $\overline{D_1} \cap \overline{D_2} = P$, being P a non-connected curve. Let us suppose that, for $i = 1, 2$, we have C^1 -functions f_i defined in D_i in such way that the piecewise function f defined as $f_i(x, y)$ if $(x, y) \in D_i$ is just continuous along the curve P , i. e., P is an ‘edge’ in the graphic of f which is unknown inside the hole of the graphic of f over H . In this work we propose a method to construct another function s_f , defined in the whole D , in such a way that:

- i) P is reconstructed inside H ;
- ii) s_f approximates f in $D - H$;
- iii) s_f interpolates f in a set of points of P ;
- iv) s_f extends the ‘shape’ of f from $D - H$ to H by respecting the edge P .

That is, s_f is a reconstruction of f which respects the ‘edge’ of its graphic over P and extends it to H . We give the basic theoretical results and we show some graphical examples to illustrate the proposed method.

References

- [1] D. Barrera, M. A. Fortes, P. González and M. Pasadas, *Filling polygonal holes with minimal energy surfaces on Powell-Sabin type triangulations*, Journal of Computational and Applied Mathematics 234 (2010), 1058–1068.
- [2] C. K. Chui and M-J. Lai, *Filling polygonal holes using C^1 -cubic triangular patches*, Computer Aided Geometric Design 17 (2000), 297–307.
- [3] M. A. Fortes, P. González, A. Palomares and M. Pasadas, *Filling holes with shape preserving conditions*, Mathematics and Computers in Simulation 118 (2015), 198–212.
- [4] J. A. Gregory and J. Zhou, *Filling polygonal holes with bicubic patches*, Computer Aided Geometric Design 11 (1994), 391–410.
- [5] M. Laghchim-Lahlou and P. Sablonnière, *C^r -finite elements of Powell-Sabin type on the three direction mesh*, Adv. in Comput. Math. 6 (1996), 191–206.
- [6] M. J. D. Powell and M. A. Sabin, *Piecewise quadratic approximations on triangles*, ACM Trans. Math. Software 3(4) (1977), 316–325.
- [7] V. Rayevskaya and L. S. Schumaker, *Multi-sided macro-element spaces based on Clough-Tocher triangle splits with applications to hole filling*, Computer Aided Geometric Design 22 (2005), 57–79.
- [8] P. Sablonnière, *Error bounds for Hermite interpolation by quadratic splines on an α -triangulation*, IMA Journal of Numerical Analysis 7, No. 4 (1987), 495–508.

A fast algorithm for computing the mock-Chebyshev interpolation

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Abstract

A very well-known example known as Runge Phenomenon published by C. Runge in 1901 is as follows: polynomial interpolation of a function f , using equidistant interpolation points on $[-1, 1]$ could diverge on certain parts of this interval even if f is analytic anywhere on the interval. Among all the techniques that have been proposed to defeat this phenomenon in the literature of approximation theory, there is the mock-Chebyshev interpolation on a grid: a subset of $(n + 1)$ points from an equispaced grid with $O(n^2)$ points chosen to mimic the non-uniform $(n + 1)$ -point Chebyshev-Lobatto grid [1].

This study suggests a fast algorithm for computing the mock-Chebyshev nodes using the distance between each of the two consecutive points. The complexity of the algorithm is $O(n)$, where $n + 1$ is the number of the Chebyshev nodes on the interval $[-1, 1]$. A discussion of bivariate generalization of the mock-Chebyshev nodes to the Padua interpolation points in $[-1, 1]^2$ is given and numerical results are also provided.

References

- [1] *J.P. Boyd, F. Xu*, Divergence (Runge Phenomenon) for least-squares polynomial approximation on an equispaced grid and MockChebyshev subset interpolation. *Appl.Math.Comput.* **210** (2009) 158–168.

Asymptotic off polar orthogonal polynomials

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Abstract

Let μ be a finite positive measure defined on the Borelian σ -algebra of \mathbb{C} , μ is absolutely continuous with respect to the Lebesgue measure $d\theta$ on $[-\pi, +\pi]$. Let us consider $\{L_n(z)\}_{n \in \mathbb{N}}$, the system of monic orthogonal polynomial with respect to μ . We introduce a new class of polynomials $\{Q_n(z)\}_{n \in \mathbb{N}}$, that we call polar polynomials associated to $\{L_n(z)\}_{n \in \mathbb{N}}$.

We aim studying this polar orthogonal polynomials on the unit circle with respect to μ . We speaking the asymptotic behavior of polar orthogonal polynomials on the unit circle with respect to μ .

References

- [1] E. MUHAMADIEV. On a certain functional equation in the algebra of polynomials with complex coefficients. Abstract and Applied Analysis, Volume 2006, Article ID 94509. pp.1–15.
- [2] Q.G.MOHAMMAD. Location of the Zeros of Polynomials. The American Mathematical Monthly, Vol. 74, No. 3 (Mar., 1967), pp. 290-292.
- [3] Q.I.RAHMAN. On the zeros of a polynomial and its derivative. Pacific Journal of Mathematics. Vol. 41, 1972, no.14. pp.1-4.

K. Image Processing

Regularization preconditioners for frame-based image deblurring

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Abstract

We are interested in fast and stable iterative regularization methods for image deblurring problems with space invariant blur. The associated coefficient matrix has a Block Toeplitz Toeplitz Blocks (BTTB) like structure depending on the boundary conditions imposed on the imaging model. In the literature, several strategies have been proposed in the attempt to define proper preconditioner for iterative regularization methods that involve such linear systems. Usually, the structure of the preconditioner is chosen Block Circulant with Circulant Blocks (BCCB) because it can be efficiently exploited by Fast Fourier Transform (FFT). Nevertheless, for ill-conditioned problems, it is well known that BCCB preconditioners cannot provide a strong clustering of the eigenvalues. Moreover, in order to get an effective preconditioner, it is crucial to preserve the structure of the coefficient matrix. On the other hand, thresholding iterative methods are recently successfully applied to image deblurring problems, exploiting the sparsity of the image in a proper wavelet domain.

Motivated by the results of recent papers [2, 3], we combine a nonstationary preconditioned iteration [1] with the modified linearized Bregman algorithm (MLBA) and proper regularization operators. Namely, the proposed algorithms are made of an outer step of a nonstationary Landweber iteration, preconditioned by a Tikhonov-type preconditioner, and an inner step of thresholding. The first one Tikhonov-type preconditioner is chosen as a matrix function of the corresponding BCCB approximation of the original square matrix operator, obtained imposing the proper boundary conditions that arise from the problem itself. The second one is chosen as a regularized version of the BCCB approximation but preserving the structure of the original square matrix operator. We prove that all the algorithms are regularizing and convergent, both in the free-noise case and in the noise case. Finally, several numerical experiments shows the consistency of our methods in terms of speed and quality of the restorations.

References

- [1] M. Donatelli, M. Hanke, *Fast nonstationary preconditioned iterative methods for ill-posed problems, with application to image deblurring*, Inverse Problems 29.9 (2013): 095008.
- [2] Y. Cao, M. Donatelli, D. Bianchi, T.Z. Huang, *Regularization preconditioners for frame-based image deblurring with reduced boundary artifacts*, SIAM Journal on Scientific Computing 38.1 (2016): B164-B189.
- [3] P. Dell'Acqua, M. Donatelli, C. Estatico, M. Mazza, *Structure preserving preconditioners for image deblurring*, Journal of Scientific Computing 72.1 (2017): 147-171.

Extrapolation methods and their applications in image reconstruction and restoration

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Abstract

In numerical analysis and in applied mathematics one has often to deal with sequences which converge slowly to their limit. Extrapolation methods can be used to accelerate the convergence of a slow converging sequence or even to sum up divergent series.

In the first part of the presentation, we will revise Wynn's ε -algorithm and the particular rules for treating isolated singularities, i.e. when two or more consecutive elements are equal or almost equal, and the more general particular rules proposed by Cordellier for treating non-isolated singularities, i.e. when more than two elements are equal. A new implementation of the generalized particular rules covering all the cases, namely singularities caused by two or more elements that are equal or almost equal, makes the algorithm more efficient.

The second part of the presentation will be devoted to applications of vector extrapolation in imaging problems. In particular, we will use the simplified topological ε -algorithm, introduced by Brezinski and Redivo-Zaglia, in order to extrapolate a sequence generated by some iterative regularization methods, commonly used for solving linear inverse problems, and we study the gain of applying extrapolation on these methods in image reconstruction and restoration problems. The numerical results illustrate the good performance of the accelerated methods compared to their unaccelerated versions and other methods.

References

- [1] C. Brezinski, M. Redivo-Zaglia, *The simplified topological ε -algorithms: software and applications*, M. Numer Algor, 74 (2017) 1237-1260.
- [2] F. Cordellier, *Démonstration algébrique de l'extension de l'identité de Wynn aux tables de Padé non normales*, in: L. Wuytack, ed., *Padé Approximation and its Applications*, volume 765 of LNM, pp. 36-60, Berlin, 1979. Springer-Verlag.
- [3] S. Gazzola, A. Karapiperi, *Image reconstruction and restoration using the simplified topological ε -algorithm*, Appl. Math. Comput., 274 (2016) 539-555.
- [4] A. Karapiperi, *Extrapolation methods and their applications in Numerical Analysis and Applied Mathematics*, PhD thesis, University of Padua, 2016.
- [5] P. Wynn, *Singular rules for certain non-linear algorithms*, BIT, 3 (1963) 175-195.

Convergent Domain Decomposition Methods for Total Variation Minimization

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Abstract

Total variation regularisation is an important tool to solve inverse imaging problems. In particular, in the last decades, in the literature, there have been introduced many different approaches and algorithms for minimizing the total variation. These standard techniques are iterative-sequentially formulated and therefore not able to solve large scale simulations in acceptable computational time. For such large problems we need to address methods that allow us to reduce the problem to a finite sequence of subproblems of a more manageable size, perhaps computed by one of the standard techniques. With this aim, we introduce domain decomposition methods for total variation minimization. The main idea of domain decomposition is to split the space of the initial problem into several smaller subspaces. By restricting the function to be minimized to the subspaces, a sequence of local problems, which may be solved easier and faster than the original problem, is constituted. Then the solution of the initial problem is obtained via the solutions of the local subproblems by glueing them together. In the case of domain decomposition for the non-smooth and non-additive total variation the crucial difficulty is the correct treatment of the interfaces of the domain decomposition patches. Due to the non-smoothness and non-additivity, one encounters additional difficulties in showing convergence of more general subspace correction strategies to global minimizers. In particular there do exist counterexamples indicating failure of splitting techniques, see e.g. [1]. Nevertheless, in this talk we propose overlapping domain decomposition algorithms for the total variation minimization problem with the guarantee of convergence to a minimizer of the original functional [2]. The analysis is based on the relation between the primal (original) total variation minimization problem and its dual formulation. To the best of our knowledge, this is the first successful approach of a domain decomposition strategy for total variation minimization with a rigorous convergent analysis in an infinite dimensional setting. We provide numerical experiments, showing the successful application of the algorithms.

References

- [1] Fornasier, M., Kim, Y., Langer, A., and Schönlieb, C. B.: Wavelet decomposition method for L_2 /TV-image deblurring, *SIAM Journal on Imaging Science*, 5 (2012), pp. 645–685.
- [2] Langer, A.: Overlapping Domain Decomposition Methods for Total Variation Denoising, preprint, 2018.

Image enhancement of 4 dimensional biomedical images with regularization.

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Abstract

It is well known that biomedical images that are used to diagnose tumors, injuries, and other defects in our bodies are optimal if they are free of blur and noise. That increases the confidence of the accuracy of the diagnosis and the use of the correct treatment. Biomedical Images however are susceptible to blur from the recording medical equipment and the motion of the patient as well as noise. As the equipment, for example the MRI scanners develop, the blurring changes but for each machine the blur can be estimated using phantoms and so it is important to keep studying them and finding more effective and efficient algorithms for image deblurring that work fast. Deblurring of the signals in addition to denoising is therefore essential for the efficient use of the signals in related applications. In our work, we apply a statistical Optimal Filtering method uses the Singular Value Decomposition of a first estimate of the blurring matrix and statistics to quantify uncertainty and to deblur the signal in an efficient and effective way. The method was originally developed for two dimensional images and is modified to be applied to higher dimensional signals. In this talk, we will present the method and discuss its effectiveness using a brain MR image.

References

V. Taroudaki and D.P. O’Leary, “Near-Optimal Spectral Filtering and Error Estimation for Solving Ill Posed Problems”, SIAM J. Sci. Comput. 37-6(2015), pp. A2947-A2968

L.

Model Reduction

Information-Based Model Reduction for Nonlinear Electro-Quasistatic Field Problems

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Abstract

We suggest a model reduction framework for transient nonlinear electro-quasistatic (EQS) field simulations of high-voltage devices that comprise strongly nonlinear electric field stress grading material. High-fidelity snapshots are obtained with the finite element method (FEM) along with implicit time quadrature (BDF1). The singular value decomposition (SVD) is employed to obtain the proper orthogonal decomposition (POD) modes [1], whilst nodes at which interpolation constraints are imposed, are selected according to absolute and relative information criteria.

Since strongly nonlinear behavior is essentially irreducible, the developed so-called maximal information refinement (MIR) strategy incorporates the indices of high-information content nodes into the interpolation index set. More precisely, at each node of the computational domain, we assign the spectral Shannon entropy [2] and the spectral Kullback-Leibler divergence [3], [4] of the electric potential. These quantities, viewed as local complexity gauges, are used as node selection criteria for interpolating nonlinear functions, given that they introduce nodes that preserve a maximum amount of information. Further, to limit the growth of the interpolation error, the resulting MIR-generated index set is complemented with indices that are selected with a greedy approach, similar to the one used as part of the discrete empirical interpolation method [5]. Our numerical investigations validated the performance of the MIR method in terms of improved accuracy, and without overloading the offline stage of the model reduction framework. We believe that the same information-theoretic / time series analysis approach requires more attention, since it exploits the high-fidelity snapshots and hence, it can be beneficial even for problems with strong nonlinearities that are distributed in a large part of the computational domain.

References

- [1] D. Schmidhäusler and M. Clemens. *Low-order electroquasistatic field simulations based on proper orthogonal decomposition*. IEEE Trans. Magn., 48(2):567–570, 2012.
- [2] C. E. Shannon and W. Weaver. *Mathematical theory of communication*. Combined Academic Publ., 1949.
- [3] S. Kullback and R. A. Leibler. *On information and sufficiency*. Ann. Math. Statist., 22(1):79–86, 1951.
- [4] I. Csiszar. *I-divergence geometry of probability distributions and minimization problems*. Ann. Probab., 3(1):146–158, 1975.
- [5] S. Chaturantabut and D. C. Sorensen. *Nonlinear model reduction via discrete empirical interpolation*. SIAM J. Sci. Comput., 32(5):2737–2764, 2010.

Sampling-free parametric model reduction of structured systems

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Abstract

We consider a parametric linear time invariant dynamical systems represented as

$$\begin{aligned} E\dot{x}(t) &= A(p)x(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned}$$

where $E, A(p) \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $C \in \mathbb{R}^{l \times n}$. We assume that $A(p)$ depends on $k \ll n$ parameters $p = (p_1, p_2, \dots, p_k)$ such that $A(p) = A_0 + U \text{diag}(p_1, p_2, \dots, p_k) V^T$, where $U, V \in \mathbb{R}^{n \times k}$ are given fixed matrices. Here $x(t) \in \mathbb{R}^n$ denotes the state variable, while $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^l$ represent, respectively, the inputs and outputs of the system.

We propose an approach for approximating the full-order transfer function $H(s; p) = C(sE - A(p))^{-1}B$ with a reduced-order model that retains the structure of parametric dependence and (typically) offers uniformly high fidelity across the full parameter range. Remarkably, the proposed reduction process removes the need for parameter sampling and thus does not depend on identifying particular parameter values of interest. In our approach the Sherman-Morrison-Woodbury formula allows us to construct a parameterized reduced order model from transfer functions of four subsystems that do not depend on parameters. In this form one can apply well-established model reduction techniques for non-parametric systems. The overall process is well suited for computationally efficient parameter optimization and the study of important system properties.

One of the main applications of our approach is for damping optimization: we consider a vibrational system described by

$$\begin{aligned} M\ddot{q}(t) + (C_{int} + C_{ext})\dot{q}(t) + Kq(t) &= Ew(t), \\ z(t) &= Hq(t), \end{aligned}$$

where the mass matrix, M , and stiffness matrix, K , are real, symmetric positive-definite matrices of order n . Here, $q(t)$ is a vector of displacements and rotations, while $w(t)$ and $z(t)$ represent, respectively, the inputs (typically viewed as potentially disruptive) and outputs of the system. Damping in the structure is modeled as viscous damping determined by $C_{int} + C_{ext}$ where C_{int} and C_{ext} represent contributions from internal and external damping, respectively. Information regarding damper geometry and positioning as well as the corresponding damping viscosities are encoded in $C_{ext} = U \text{diag}(p_1, p_2, \dots, p_k) U^T$ where $U \in \mathbb{R}^{n \times k}$ determines the placement and geometry of the external dampers.

The main problem is to determine the best damping matrix that is able to minimize influence of the disturbances, w , on the output of the system z . We use a minimization criteria based on the \mathcal{H}_2 system norm. In realistic settings, damping optimization is a very demanding problem. We find that the parametric model reduction approach described here offers a new tool with significant advantages for the efficient optimization of damping in such problems.

Poster Session

A Policy Iteration Algorithm for Pricing the American Option

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Abstract

The pricing of American option is a challenging problem especially for high dimensional cases, since finite difference and binomial tree techniques become impractical in presence of multiple factors (curse of dimensionality). Even in the one factor case, the problem is still challenging since the value of the option is determined from the solution of Black-Scholes equation on the free boundary condition (the optimal exercise boundary).

We present an alternative way for pricing American style options based on the policy iteration dynamic programming algorithm which leads to monotonically increasing value functions. We mention that a policy in an American option can be specified by an exercise boundary, the option being exercised at the instance the underlying asset first reaches the boundary. Under reasonable assumptions, we show that the algorithm converges quadratically, and we present a numerical implementation which indeed exhibits fast convergence.

References

- [1] E. Mageirou, P. Vassalos, and N. Barakitis. *A Policy Iteration Algorithm for the American Put Option*. Preprint, DOI: 10.13140/RG.2.2.29578.57283

Weighting Assessment of Vulnerability Index Parameters of Reinforced Concrete Constructions

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Abstract

Seismic vulnerability assessment of reinforced concrete (RC) existing structures can be performed through the use of reliable tools. This is in order to reduce damages in case of an earthquake event.

In this paper, a vulnerability index method has been applied to this type of building for the Algerian case according to the national seismic regulations (RPA), by identifying the most important parameters that have an influence on the seismic behavior of such structures. Weighting factors are then assigned to each parameter in order to evaluate the vulnerability index, which allows classifying each assessed structure according to a proposed classification. The weighting factors were estimated by a dynamic analysis using ten seismic records. Lastly, this method was implemented using a developed numerical code and performed using several examples to show its efficiency.

Keywords: Vulnerability index, reinforced concrete construction, finite element, dynamic analysis, earthquake, damage.

Modeling and simulation of eddy current testing of aeronautical tubes

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Abstract

Eddy current testing can be used such as a perfect tool to characterize defects in materials. In particular, on fields of advanced industry those require a maximal safety (aeronautics, aerospace, nuclear...). However, the sensitivity of the characterization process is highly dependent on the probe choice and the operation frequency.

In this context, it is necessary to identify and control cracks in metallic tubes used in aeronautic, especially those prepared by aluminum.

Detection of axial cracks in tubes continues to be a major challenge in aeronautical studies. The general idea is to provide a theoretical and computational framework for the efficient and approximate treatment of three-dimensional electromagnetic problem, i.e. the simulation of cracks in the presence of an eddy-current probe of arbitrary configuration.

To do this, equations of electromagnetic field on the surface of a conductor of cylindrical shape are developed and then the FEM is applied to its solution.

The developed code has a graphical user interface and can be used for fast computation and plotting of various impedance diagrams.

Subspace iteration method for generalized singular values

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Abstract

It's well known that the Singular Values Decomposition (SVD) is useful in many applications such as low rank approximation, data reductions, identification of the best approximation of the original data points using fewer dimensions. It's also a useful tool for computation of eigenvalues of matrix $A^T A$ without explicitly forming the matrix product. The Generalized Singular Values Decomposition (GSVD) of the pair (A, B) is also a useful tool for computation of the generalized eigenvalues of the symmetric pencil $A^T A - \lambda B^T B = 0$. The generalized singular values of the pair (A, B) are nothing but the square roots of generalized eigenvalues of the symmetric eigenproblem $A^T A - \lambda B^T B = 0$. We present available methods to compute the largest generalized singular values and vectors using iterative subspace-like method. The new approach gives considerably better efficiency compared to the matlab function . Numerical examples show the effectiveness of the presented method.

Keywords: singular value, generalized singular values problem, matrix pair, power method, ...

References

- [1] A. H. BENTBIB and A.KANBER *Block Power Method for SVD Decomposition* , (2007). An. tiin. Univ. Ovidius” Constana, Ser. Mat. 23, No. 2, 45-58 (2015).
- [2] G. H. GOLUB and C. F. VAN LOAN, *Matrix Computation* , 4th Edition , 2013.
- [3] M. E. HOCHSTENBACH. *A Jacobi-Davidson type method for the generalized singular value problem*. Linear Algebra and its Applications Volume 431, Issues 3-4, 15 July 2009, Pages 471-487.
- [4] SABINE VAN HUFFEL, *Iterative algorithms for computing the singular subspace of a matrix associated with its smallest singular values* , Linear Algebra and its Applications Volumes 154156, August-October 1991, Pages 675-709.
- [5] C. C. PAIGE. *Computing the Generalized Singular Value Decomposition*. SIAM J. Sci. and Stat. Comput., 7(4), 1126-1146. (21 pages)
- [6] A. H. REFAHI SHEIKHANI and S. KORDROSTAMI *New iterative methods for generalized singular-value problems*. Mathematical Sciences December 2017, Volume 11, Issue 4, pp 257-265.
- [7] X. YU. *Generalized Krylov subspace methods with applications*. August, 2014.

Metric regularity of composition multifunctions

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Abstract

The aim of this paper is to give a metric regularity theorem for composition of set-valued mappings between metric spaces involving a new concept of composition stability.

Keywords. Set-valued mapping, Metric regularity, Composition multifunction, Composition stability.

Mathematics Subject Classification (2000). 90C25, 49M45, 65C25.

References

- [1] M. Durea, H. V. Ngai, H.T. Nguyen and R. Strugariu, Metric regularity of composition set-valued mapping: metric setting and coderivative conditions. *Journal of Mathematical Analysis and Applications*. Volume 412, Issue 1, 1 April 2014, Pages 41–62.
- [2] M. Durea, R. Strugariu, Chain rules for linear openness in general Banach spaces. *SIAM Journal on Optimization*, 22 (2012), 899–913.
- [3] M. Durea, R. Strugariu, Chain rules for linear openness in metric spaces and applications, *Mathematical Programming, Serie A*, DOI: 10.1007/s10107-012-0598-8.
- [4] M. Durea, H.T. Nguyen, R. Strugariu, Metric regularity of epigraphical multivalued mappings and applications to vector optimization, *Mathematical Programming, Serie B* DOI 10.1007/s10107-013-0665-9.
- [5] A. D. Ioffe. Metric regularity, fixed points and some associated problems of variational analysis. *J. Fixed Point Theory Appl.* 15 (2014) 67–99 DOI 10.1007/s11784-014-0186-z.
- [6] A. D. Ioffe, Regularity on a fixed set. *SIAM J. Optim.* 21 (2011), 1345–137.
- [7] A. Ioffe, Metric regularity: Theory and applications - a survey. (2015)
- [8] H.V. Ngai, M. Théra, Error bounds in metric spaces and application to the perturbation stability of metric regularity, *SIAM Journal on Optimization*, 19 (2008), 1–20.
- [9] H. V. Ngai, H.T. Nguyen and M. Théra, Metric Regularity of the Sum of Multifunctions and Applications. 2013. hal-00819047.

Mathematical Structures defined by Identities III

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Abstract

In this work we extend the theory (formal part only) of algebras with one binary operation [1], to algebras with several operations of any arity. We briefly outlined our ideas of generalizing the method of tableaux to algebras with several operations

$$V_1(x_1, x_2, \dots, x_{a_1}), V_2(x_1, x_2, \dots, x_{a_2}), \dots, V_k(x_1, x_2, \dots, x_{a_k})$$

satisfying axiomatically defined identities and indicated the way of how to proceed. The project is now carried out. The technique applied is the same as in Formal Part of [1]. The crucial fact that the number $I_n^{V_1 V_2 \dots V_k}$ of formally reducible identities can be calculated by exactly the same method used for $I_n^{V_1} (= I_n)$ seems to hold true. Algebras with only binary operations are discussed. For algebras with two binary operations $V(x, y)$ and $W(x, y)$ the proof is given in detail. Algebras with operations of any arity can be treated by reduction to a well defined set of algebras with binary operations.

Research and exposition of the general theory are impeded by problems of construction and inspection of the tableaux T_n whenever n is greater than 3. This is due to the fast growth of the Catalan numbers ($S_n \sim \frac{4^n}{\pi^{\frac{1}{2}} n^{\frac{3}{2}}}$) and their generalizations, let alone problems of printing and publication. Programs designed to seek the structures resulting from a given identity failed after a few steps (blow-ups). Exposition therefore is limited to illustrate the theory on the worked example of tableau T_3 .

Still, the concrete new findings reached in this case corroborate further our fundamental thesis that there is a scarcity of existing mathematical structures in the sense that the frequency of irreducible identities goes to zero with increasing n . Seen historically, this also explains why mathematics, in the course of time, has developed the way it did with associativity $V(V(x, y), z) = V(x, V(y, z))$, the simplest structure, reigning supreme over the mathematical landscape. All other essential mathematical structures, found or created by research such as e.g. Groups, Fields, Vector Spaces, Lie Algebras, etc, ... include in their axiom system (signature) at least one binary operation obeying the law of associativity. We conclude with a note on the connection with Formal Languages

References

- [1] Petridi, C. M., Krikelis, P. B., Mathematical Structures defined by Identities, arXiv:math/0110333v1 [math.RA] 31 Oct 2001.
- [2] Petridi, C. M., Mathematical Structures defined by Identities II, arXiv:1009.1006v1 [math.RA] 6 Sep 2010.
- [3] Petridi, C. M., Mathematical Structures defined by Identities III, arXiv:1506.00798v1 [math.LO] 2 Jun 2015.

Implementation Techniques for Non-Symmetric Real Toeplitz Systems by Preconditioned GM-RES Method

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Abstract

In this poster we show some implementation techniques of real non-symmetric and non-definite Toeplitz systems $T_n(f)x = b$ by Preconditioned Generalized Minimum Residual (PGMRES) method. Toeplitz matrices have the same entries along their diagonals and are generated from the Fourier coefficients of a 2π -periodic generating function or symbol f . Such systems appear in various Mathematical Topics: Differential and Integral equations, Mechanics, Fluid Mechanics and in Applications: signal processing, image processing and restoration, time series, and queueing networks.

We follow the results of the paper presented by Noutsos in NASCA2018 conference to show techniques of constructing efficient preconditioners, especially for illconditioned problems.

Moreover, we will present techniques of solving such systems by PGMRES method when the generating function is unknown. We propose techniques of approximating f from the entries of the Toeplitz matrix, using Fourier expansions or Rayleigh quotients. We present a procedure which estimates the points where f has roots as well as the multiplicity of them. Taking this information into account we construct efficient preconditioners.

The efficiency of the proposed techniques are shown by various numerical experiments.

Timetable

Talks will be given at two parallel sessions.

Monday 2 July

Morning

08:00–08:40	Registration	
08:40–09:00	Opening Remarks	
09:00–09:45	Vassilios Dougalis	
	<i>Session A1.</i>	<i>Session B1.</i>
	<i>Numerical PDEs</i>	<i>Numerical Linear Algebra</i>
09:50–10:15	Angel Duran	Sang-Hyup Seo
10:15–10:40	Eduardo Abreu	Achiya Dax
10:40–11:05	Hyam Abboud	Koen Ruymbeek
11:05–11:30	Coffee Break	
11:30–12:00	Thomas Richard , Maple presentation, Part 1	
	<i>Session K1.</i>	<i>Session E1.</i>
	<i>Image Processing</i>	<i>ODEs, PDEs and Integral Equations</i>
12:00–12:25	Anna Karapiperi	Patricia Díaz de Alba
12:25–12:50	Andreas Langer	Bor Plestenjak
12:50–13:15	Davide Bianchi	Cristina Anton
13:15–13:40	Viktoria Taroudaki	Robert Kaiser

Afternoon

17:00–17:45	Hassane Sadok	
17:45–18:00	Refreshments	
	<i>Session B2.</i>	<i>Session G1.</i>
	<i>Numerical Linear Algebra</i>	<i>Linear Algebra</i>
18:00–18:25	Joab Winkler	Jie Meng
18:25–18:50	Andreas Stathopoulos	Leila Lebtahi
18:50–19:15	Panayiotis Psarrakos	Ji Eun Kim
19:20–19:50	Poster Blitz	

Evening

21:00	Welcome Reception
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Tuesday 3 July

Morning

09:00–09:45	Dario Bini <i>Session B3.</i> <i>Numerical Linear Algebra</i>	<i>Session I1.</i> <i>Optimization</i>
09:50–10:15	Dimitrios Noutsos	Hamadi Ammar
10:15–10:40	Sean Hon	Denis Hamad
10:40–11:05	Il’In Valery	George Miminis
11:05–11:30	Coffee Break	
11:30–12:00	Thomas Richard , Maple presentation, Part 2	
	<i>Session D1.</i> <i>Data Assimilation</i>	<i>Session A2.</i> <i>Numerical PDEs</i>
12:00–12:25	Azzouz Dermoune	Orestis Vantzios
12:25–12:50	Pietro Dell’Acqua	Lyudmila Vshivkova
12:50–13:15	Paulius Palevičius	Ivan Yotov
13:15–13:40	Victor Shutyaev	Paris Vassalos

Afternoon

17:00–17:45	Michele Benzi	
17:45–18:00	Refreshments	
	<i>Session B4.</i> <i>Numerical Linear Algebra</i>	<i>Session A3.</i> <i>Numerical PDEs</i>
18:00–18:25	Paraskevi Fika	Ismail Merabet
18:25–18:50	Vasileios Georgiou	Jean-Marc Gratien
18:50–19:15	Mohamed El Guide	Sylwester Arabas
19:20–19:50	Marilena Mitrouli	Abdelmalek Zine

Wednesday 4 July

Morning

09:00–09:45	Michael Tsatsomeros <i>Session G2.</i> <i>Linear Algebra</i>	<i>Session H1.</i> <i>Inverse Problems</i>
09:50–10:15	Dimitrios Triantafyllou	Natalia Lezina
10:15–10:40	Aikaterini Aretaki	Ioannis Arkoudis
10:40–11:05	Plamen Koev	Alexey Penenko
11:05–11:30	Dimitrios Christou	Stefania Zoi
11:30–12:00	Coffee Break	
	<i>Session B5.</i> <i>Numerical Linear Algebra</i>	<i>Session L1.</i> <i>Model Reduction</i>
12:00–12:25	Yuli Eidelman	Zoran Tomljanovic
12:25–12:50	Zvonimir Bujanovic	Fotios Kasolis
12:50–13:15	Daan Camps	Alicia Herrero Debón

Afternoon

The Conference Excursion to Methoni

Thursday 5 July

Morning

09:00–09:45	Sotirios Notaris <i>Session C1.</i> <i>Regularization Methods</i>	<i>Session B6.</i> <i>Numerical Linear Algebra</i>
09:50–10:15	Alessandro Buccini	Franck Dufrenois
10:15–10:40	Claudio Estatico	Sasa Stanko
10:40–11:05	Paraskevi Roupá	Fuminori Tatsuoka
11:05–11:30	Coffee Break <i>Session J1.</i> <i>Approximation Theory</i>	<i>Session A4.</i> <i>Numerical PDEs</i>
11:30–11:55	Paola Boito	Francisco Bernal
11:55–12:20	B. Ali Ibrahimoglu	Yassin Belkourchia
12:20–12:45	Miguel Ángel Fortes	Marta Paliaga
12:45–13:10	Abdelhamid Rehouma	Yuri Laevsky

Afternoon

	<i>Session B7.</i> <i>Numerical Linear Algebra</i>	<i>Session D2.</i> <i>Data Assimilation</i>
17:00–17:25	Hessah Alqahtani	Anna Concas
17:25–17:50	Oussama Abidi	Natalia Zakharova
17:50–18:15	Fatemeh Panjeh Ali Beik	Eugene Parmuzin
18:15–18:40	Ahmed Salam	Tomoaki Okayama
18:40–18:55	Refreshments	
18:55–19:40	Lothar Reichel	

Evening

21:00	Conference Dinner
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Friday 6 July

Morning

09:00–10:00	Lecture course by Paul Van Dooren: <i>Linearizations of polynomial and rational matrices</i>
10:00–11:00	Lecture course by Yousef Saad: <i>Dimension reduction techniques: Algorithms and applications</i>
11:00–11:30	Coffee Break
11:30–12:30	Lecture course by Petros Drineas: <i>RandNLA: Randomization in Numerical Linear Algebra</i>
12:30	Closing Remarks

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