

# Error estimate for the Gauss quadrature formula: The Gauss-Kronrod vs the anti-Gaussian approach

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## Abstract

It is well known that a practical error estimator for the Gauss quadrature formula is by means of the corresponding Gauss-Kronrod quadrature formula developed by Kronrod in 1964. However, recent advances show that Gauss-Kronrod formulae fail to exist, with real and distinct nodes in the interval of integration and positive weights, for several of the classical measures. An alternative to the Gauss-Kronrod formula, as error estimator for the Gauss formula, is the anti-Gaussian and the averaged Gaussian quadrature formulae presented by Laurie in 1996. These formulae always exist and enjoy the nice properties that, in several cases, Gauss-Kronrod formulae fail to satisfy. After a brief overview of the Gauss-Kronrod, the anti-Gaussian and the averaged Gaussian formulae, we try to answer the question whether there are measures for which the Gauss-Kronrod and the averaged Gaussian formulae coincide, thus leading to the same error estimate for the Gauss formula. It is quite remarkable that this is true for a certain, quite broad, class of measures, which is described in terms of the three-term recurrence relation that the corresponding orthogonal polynomials satisfy.

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