# Generalized Block Anti-Gauss Quadrature Rules

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## Abstract

Golub and Meurant describe how pairs of Gauss and Gauss–Radau quadrature rules can be applied to determine inexpensively computable upper and lower bounds for certain real-valued matrix functionals defined by a symmetric matrix. However, there are many matrix functionals for which their technique is not guaranteed to furnish upper and lower bounds. In this situation, it may be possible to determine upper and lower bounds by evaluating pairs of Gauss and anti-Gauss rules. Unfortunately, it is difficult to ascertain whether the values determined by Gauss and anti-Gauss rules bracket the value of the given real-valued matrix functional. Therefore, generalizations of anti-Gauss rules have recently been described, such that pairs of Gauss and generalized anti-Gauss rules may determine upper and lower bounds for real-valued matrix functionals also when pairs of Gauss and (standard) anti-Gauss rules do not. The available generalization requires the matrix that defines the functional to be real and symmetric. The present paper extends generalized anti-Gauss rules in several ways: The real-valued matrix functional may be defined by a nonsymmetric matrix. Moreover, extensions that can be applied to matrix-valued functions are presented. Estimates of element-wise upper and lower bounds then are determined. Finally, modifications that yield simpler formulas are described.