A Householder-based algorithm for Hessenberg-triangular reduction

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Abstract

Reducing the matrix pair (A, B) to Hessenberg-triangular form is an important and time-consuming preprocessing step when computing eigenvalues and eigenvectors of the pencil $A - \lambda B$ by the QZ-algorithm. Current state-of-the-art algorithms for this reduction are based on Givens rotations, which limits the possibility of using efficient level 3 BLAS operations, as well as parallelization potential on modern CPUs. Both of these issues remain even with partial accumulation of Givens rotations [1], implemented, e.g., in LAPACK.

In this talk we present a novel approach for computing the Hessenberg-triangular reduction, which is based on using Householder reflectors. The key element in the new algorithm is the lesser known ability of Householder reflectors to zero-out elements in a matrix column even when applied from the right side of the matrix [2, 3]. The performance of the new reduction algorithm is boosted by blocking and other optimization techniques, all of which permit efficient use of level 3 BLAS operations. We also discuss measures necessary for ensuring numerical stability of the algorithm. While the development of a parallel version is future work, numerical experiments already show benefits of the Householder-based approach compared to Givens rotations in the multicore computing environment.

References

[1] B. Kågström, D. Kressner, E. S. Quintana-Ortí, G. Quintana-Ortí, *Blocked algorithms for the reduction to Hessenberg-triangular form revisited*, BIT, 48(3):563–584, 2008.

[2] D. S. Watkins, *Performance of the QZ algorithm in the presence of infinite eigenvalues*, SIAM J. Matrix Anal. Appl., 22(2):364–375, 2000.

[3] B. Kågström, D. Kressner, *Multishift variants of the QZ algorithm with aggressive early deflation*, SIAM J. Matrix Anal. Appl., 29(1):199–227, 2006.