One approach to the numerical simulation of the filtration problem in the presence of wells with given total flow rates

Yu.M. Laevsky[1]

[1] *Institute of Computational Mathematics and Mathematical Geophysics,SB RAS*
Novosibirsk State University

Abstract

First, we discuss an elliptic boundary-value problem, describing a filtration of single-phase liquid, in the presence of wells of relatively small diameters, on which the integral flow rates of the fluid are given at constant but unknown pressures. As it is known, the solution of such a problem is reduced to solving a set of auxiliary problems according to the number of wells. We propose an alternative approach that consists of solving only one problem in a mixed weak formulation. In this case, a mixed formulation in the form of a system of equations of the first order makes it possible to carry out the extension of the solution by a constant into the wells, and the approach can be treated as a sort of fictitious domain method. Numerical implementation is based on a mixed finite element method with the Raviart-Thomas basis functions of the least degree. Error estimates are obtained and the results of computational experiments are presented.

Second, we expand the approach proposed to the non-stationary problem of a two-phase liquid filtration, and to the 3D filtration problem with wells parallel to one of the coordinate axes. In this case, the total velocity and pressure satisfy the quasi-stationary system of saddle-type equations discussed above. The non-stationary equation for saturation is also obtained using the fictitious domain method. The main feature of the approach proposed is that pressure and saturation belong to the same functional space. This is achieved by setting the phase velocity as the orthogonal projection of the total velocity multiplied by the relative permeability of the phase to the subspace of vector-functions with square summable divergence. Similar arguments are given for approximation of finite-dimensional subspaces. In the case of 3D problem, the solution in the direction of the wells is much smoother, and, therefore, it makes sense to introduce fluxes only in the directions orthogonal to the wells. As a result, we arrive at a mixed weak formulation, which is anisotropic in the sense of smoothness. For the numerical implementation a combination of the finite element method and the mixed finite element method is used, which we call the anisotropic mixed finite element method.
