# Sensitivity of the optimal solution of variational data assimilation problems

V. Shutyaev[1,2], F.-X. Le Dimet[3], E. Parmuzin[1,2]

[1] *Institute of Numerical Mathematics Russian Academy of Sciences, Moscow, Russian Federation*
[2] *Moscow Institute of Physics and Technology, Dolgoprudny, Moscow Region, Russian Federation*
[3] *LJK, Université Grenoble Alpes, France*

## Abstract

The problem of variational data assimilation for a nonlinear evolution model is formulated as an optimal control problem to find the unknown parameters of the model. We study the problem of sensitivity of the optimal solution via variational data assimilation with respect to observation errors. On the basis of relations between the error of the optimal solution and the errors of observational data through the Hessian of the cost functional, the algorithms are developed and justified for calculating the coefficients of sensitivity as the norms of the response operators occurring in the equations for errors. A numerical study of the sensitivity of the optimal solution on the example of the problem of variational data assimilation of sea surface temperature to restore the heat flows for the model of thermodynamics is presented. Numerical examples for data assimilation in the Baltic Sea dynamics model are given. This work was carried out within the Russian Science Foundation project 14-11-00609 (numerical experiments) and the project 18-01-00267 of the Russian Foundation for the Basic Research.

# References

[1] Gejadze I., Le Dimet F.-X., Shutyaev V. On analysis error covariances in variational data assimilation. SIAM J. Sci. Comput., 2008, v.30, no.4, pp. 1847–1874.

[2] Gejadze I., Le Dimet F.-X., Shutyaev V. On optimal solution error covariances in variational data assimilation problems. J. Comp. Phys., 2010, v.229, pp. 2159–2178.

[3] Gejadze I., Shutyaev V.P., Le Dimet F.-X. Analysis error covariance versus posterior covariance in variational data assimilation. Q. J. R. Meteorol. Soc., 2013, v.139, pp. 1826–1841.

[4] Gejadze, I.Yu., Shutyaev, V. On gauss-verifiability of optimal solutions in variational data assimilation problems with nonlinear dynamics. J. Comp. Phys., 2015, v.280, pp.439–456.

[5] Le Dimet, F.-X., Shutyaev, V., Parmuzin, E.I. Sensitivity of functionals with respect to observations in variational data assimilation. Russ. J. Numer. Anal. Math. Modelling, 2016, v.31, no.2, pp.81–91.

[6] Shutyaev, V., Gejadze, I., Vidard, A., and Le Dimet, F.-X. Optimal solution error quantification in variational data assimilation involving imperfect models. Int. J. Numer. Meth. Fluids, 2017, v.83, no.3, pp.276–290.

[7] Gejadze, I., Oubanas, H., and Shutyaev, V. Implicit treatment of model error using inflated observation-error covariance. Q.J.R. Meteorol. Soc., 2017, v.143, pp.2496–2508.