On the two-level iterative methods in the Krylov subspaces

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Abstract

We consider different approaches to constructing of the two - level iterative procedures in the Krylov subspace for a high-performance solution of the large systems of linear algebraic equations (SLAEs) with non-symmetric, in general, sparse matrices which arise in finite element or finite volume methods (FEM or FVB) for approximation of the multi-dimensional boundary value problems with real data on the non-structured grids, when implementating on the multi-processor computers (MPC) with distributed and hierarchical shared memory. The parallel algorithms are based on the additive domain decompostion methods (DDM) , with parametrized overlapping of subdomains and various interface conditions at the internal bounda- ries, in the Krylov subspaces. At the upper level, the block multi-preconditioned semi-conjugate direction methods (MP-SCD) are applied which realize semi-conjugate gradient or residual (MP-SCG or MP-SCR) algorithms in particular cases. These algorithms are equivalent, in a sense, to FOM and GMRES methods respectively. The acceleration of the external iterative process is provided by means of the coarse grid correction, or the deflation procedure, or the low - rank approximation of the original matrix, as well as by the Sonneveld approaches. In the case of reduced recursions, the acceleration of restarted iterations is attained by the least squares approaches, The simultaneous solutions of the auxiliary sub-systems in subdomains at the low -level of the computational process are implemented by means of either direct or iterative algorithms. The parallelization of the two - level iterative processes is based on the hybrid programming using MPI and multi-thread technologies. The performance and efficiency of the proposed algorithms, which are implemented in the library KRYLOV, are demonstrated on the results of numerical experiments for a representative set of test problems.