New conformal map for the Sinc approximation for exponentially-decaying functions over the semi-infinite interval

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Abstract

The Sinc approximation is a highly-efficient approximation formula for analytic functions expressed as

$$F(x) \approx \sum_{k=-M}^{N} F(kh)\,\mathrm{sinc}(x/h-k), \quad x \in (-\infty,\infty),$$

where $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. This approximation gives exponential convergence if $|F(x)|$ decays exponentially as $x \to \pm\infty$. Here, we should also note that the target interval to be considered is the infinite interval $(-\infty,\infty)$, and accordingly F should be defined over the infinite interval. If the function to be approximated decays exponentially but is defined over the semi-infinite interval $(0,\infty)$, for example $f(t) = \sqrt{t}\,\mathrm{e}^{-t}$, Stenger [1] proposed to employ a conformal map

$$t = \psi(x) = \mathrm{arcsinh}(\mathrm{e}^{x}),$$

by which the transformed function $f(\psi(x))$ is defined over $(-\infty,\infty)$ and decays exponentially as $x \to \pm\infty$. However, conformal map performing such a role is not unique; if we employ another conformal map, the convergence rate may be improved. In fact, in the area of numerical integration, improvement of the convergence rate has been reported [2, 3] by replacing the conformal map $t = \psi(x)$ with

$$t = \phi(x) = \log(1+\mathrm{e}^{x}).$$

Motivated by the fact, this study proposes to combine the Sinc approximation with $t = \phi(x)$ instead of $t = \psi(x)$. A computable error bound for the proposed approximation formula is also given.

References

[1] F. STENGER: *Numerical Methods Based on Sinc and Analytic Functions*, Springer-Verlag, New York, 1993.

[2] T. OKAYAMA and K. MACHIDA: Error estimate with explicit constants for the trapezoidal formula combined with Muhammad–Mori's SE transformation for the semi-infinite interval, *JSIAM Letters*, Vol. 9 (2017), 45–47.

[3] R. HARA and T. OKAYAMA: Explicit error bound for Muhammad–Mori's SE-Sinc indefinite integration formula over the semi-infinite interval, *Proceedings of the 2017 International Symposium on Nonlinear Theory and its Applications*, 2017, 677–680.