MPDATA Meets Black-Scholes: Derivative Pricing as a Transport Problem

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Abstract

MPDATA stands for Multidimensional Positive Definite Advection Transport Algorithm. The iterative, explicit-in-time algorithm was introduced in [1] as a robust numerical scheme for atmospheric modelling applications. Extensions and generalisations of MPDATA continuously developed over the years constitute a family of numerical schemes offering high-order, sign-preserving and non-oscillatory solutions for transport problems (for a review, see e.g. [2], recent developments include third-order accurate formulation [3]). There is a multitude of documented applications of MPDATA across diverse domains. In the present work we demonstrate applicability of the algorithm for solving PDEs arising in financial derivative instrument pricing.

We present a generalisable framework for solving Black-Scholes-type equations by first transforming them into advection-diffusion problems, and numerically integrating using an iterative explicit finite-difference approach, in which the Fickian term is represented as an additional advective term. Leveraging this mathematical equivalence between Black-Scholes-type models and transport models, we detail applications of MPDATA to numerically reproduce the analytical solution of a celebrated benchmark problem — the Black-Scholes formula for pricing of European options — and to numerically solve the associated free boundary problem arising in the valuation of American options. These results are used for convergence analysis.

Presented work is based on [4]. Numerical solutions are obtained using libmpdata++ [5].

References

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