Convergence of a Modified Newton Method for a Matrix Polynomial Equation Arising in Stochastic Problems

Sang-Hyup Seo[1], Jong-Hyeon Seo[2], Hyun-Min Kim[3]

[1]Department of Mathematics, Pusan National University, Busan, 46241, Republic of Korea
[2]Department of Mathematics, Pusan National University, Busan, 46241, Republic of Korea
[3]Department of Mathematics, Pusan National University, Busan, 46241, Republic of Korea

Abstract

We consider the Newton iteration of a matrix polynomial equation

$$P(X) = \sum_{k=0}^{n} A_k X^k = A_n X^n + A_{n-1} X^{n-1} + \cdots + A_1 X + A_0 = 0, \tag{1}$$

which arises in stochastic problem. The elementwise minimal nonnegative solution S of (1) can be obtained using Newton's method with the zero initial value if the equation has the solution. Moreover, the convergence rate of the iteration is quadratic if P'_S, the Fréchet derivative at S, is nonsingular. If P'_S is singular, the convergence rate is at least linear. But, for any $\varepsilon > 0$, there exists an integer $i_0 > 0$ such that $\{X_i - S\}_{i=i_0}^{\infty}$ is in the ε-neighborhood of an one-dimensional space. Then, with a modified Newton method

$$X_{i+1} = X_i - \lambda P'^{-1}_{X_i}(P(X_i)), \tag{2}$$

we can reduce the iteration number. In the representation, we will give the proofs and some numerical experiments of this abstract.

References

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