On a Collocation-quadrature Method for the Singular Integral Equation of the Notched Half-plane Problem

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Abstract

This contribution deals with a notched half plane problem of two-dimensional elasticity theory, which considers a straight crack of normalized length 2 perpendicular to and ending at the boundary of the elastic half plane. The problem can be modelled by a hypersingular integral equation, the solution of which is the crack opening displacement. For the numerical solution of this equation we propose polynomial collocation-quadrature methods, which look for an approximation of the derivative of the crack opening displacement. This derivative is the solution of a Cauchy singular integral equation with additional fixed singularities, which is given by the equation

$$\frac{1}{\pi}\int_{-1}^{1}\left[\frac{1}{y-x}+\mathbf{h}\left(\frac{1+x}{1+y}\right)\frac{1}{1+y}\right]v'(y)\,dy=f(x)\,,\quad x\in(-1,1)\,,$$

where the right hand side $f:[-1,1]\longrightarrow\mathbb{C}$ is smooth and where

$$\mathbf{h}(t)=-\frac{1}{1+t}+\frac{6t}{(1+t)^2}-\frac{4t^2}{(1+t)^3}\,.$$

By using C*-algebra techniques as well as results from a previous work [1], we present necessary and sufficient conditions for the stability of the collocation-quadrature methods as well as numerical results. In contrast to former works (cf. [1] and [2]) our results are based on a new approach, which takes into account the "natural" asymptotic of the solution at the endpoints of the integration interval and for which until now no criterion for stability is known.

References

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