Accurate and Efficient Traces of Beta–Wishart Matrices

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Abstract

The eigenvalues of a Wishart random matrix and its trace have long been used in multivariate statistical analysis for a variety of analyses and applications [4]. This class was recently generalized to any $\beta > 0$ to obtain the class of Beta–Wishart matrices of which the classical real, complex, and quaternion cases correspond to $\beta = 1, 2$, and 4, respectively [1, 2].

The only known expressions for the eigenvalues and the trace, however, are in terms of infinite series of Jack functions, and in particular, the hypergeometric function of a matrix argument. These series are notoriously slow to converge and have been a computational challenge for decades despite recent progress [3]. The main issue is the exponential number of terms in (a finite truncation of) the expansion of hypergeometric function as a series of Jack functions.

We will present new expressions and a new algorithm for computing the density and distribution of the trace of a Beta–Wishart matrix which is linear in both the size of the matrix *and* the degree of the truncation. This complexity is optimal. Additionally, our new algorithm is subtraction-free, which means that the results will be computed to high relative accuracy in the presence of roundoff errors in floating point arithmetic.

Additionally, we will present new results that allow the computation of the density of the largest eigenvalue of a Beta–Wishart matrix order of magnitude faster than previous results.

References

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