Generalized Matrix Functions: Theory and Computation

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Abstract

Generalized matrix functions were introduced in 1973 by Hawkins and Ben-Israel [4] with the aim of extending the notion of matrix function to rectangular matrices. If A has rank r and $A = U_r \Sigma_r V_r^*$ is a compact SVD of A, then $f^\diamond(A) := U_r f(\Sigma_r) V_r^*$ defines a generalized matrix function of A, provided that the real-valued scalar function f is defined on the singular values of A. Generalized matrix functions arise naturally in several applications, ranging from the solution of rank-constrained matrix minimization problems to the analysis of directed networks. When A is large, computing its SVD becomes prohibitively expensive. Hence, approximation methods are required for computing quantities related to generalized matrix functions, such as $f^\diamond(A)\mathbf{v}$ for a given vector $\mathbf{v}$. This talk will discuss theoretical aspects of generalized matrix functions, such as the preservation of structural properties present in A [3], as well as approximation methods based on Golub–Kahan bidiagonalization [1] and on Chebyshev polynomial interpolation [2].

Collaborators on this project include Francesca Arrigo, Caterina Fenu, Ru Huang, Jared Aurentz, Anthony Austin, and Vassilis Kalantzis.

References

[1] F. Arrigo, M. Benzi, and C. Fenu, *Computation of generalized matrix functions*, SIAM Journal on Matrix Analysis and Applications, 37 (2016), 836–860.

[2] J. L. Aurentz, A. P. Austin, M. Benzi, and V. Kalantzis, *Stable computation of generalized matrix functions via polynomial interpolation*, Preprint, December 2017. Submitted.

[3] M. Benzi and R. Huang, *Structural properties preserved by generalized matrix functions*, in preparation.

[4] J. B. HAWKINS AND A. BEN–ISRAEL, *On generalized matrix functions*, Linear and Multilinear Algebra, 1 (1973), pp. 163–171.