Efficient calculating the selected eigenvalues of parametrized matrices

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Abstract

In applications of linear algebra including nuclear physics and structural dynamics, there is a need to deal with uncertainty in the matrices. We focus on matrices that depend on a set of parameters ω. In this talk, we are interested in the smallest eigenvalue of a large scale generalized eigenvalue problem with symmetric positive definite matrices since in that case the eigenvalues are real and the eigenvectors satisfy some orthogonality properties. If ω can be interpreted as the realisation of random variables, one may be interested in statistical moments of the smallest eigenvalue. In order to obtain statistical moments, we need a fast evaluation of the eigenvalue as a function of ω. Since calculating this is costly for large matrices, we are looking for a small parametrized eigenvalue problem, whose smallest eigenvalue makes a small error with the smallest eigenvalue of the large eigenvalue problem.

The advantage, in comparison with a global polynomial approximation (on which, e.g., the polynomial chaos approximation relies), is that we do not suffer from the possible non-smoothness of the smallest eigenvalue. The small scale eigenvalue problem is obtained by projection of the large scale problem. The idea is to filter out the subspace which is not needed for determining the smallest eigenvalue. We developed projection methods based on the principle that an eigenvalue of the projected eigenvalue problem is also an eigenvalue of the large eigenvalue if the eigenvector is present in the associated subspace. By this we interpolate between the points for which the eigenvector are added to the subspace.

Besides this, we will also talk about the choice of interpolation points, how to efficiently check the overall error and compare our method with a polynomial approximation. Numerical examples from structural dynamics are given.
