Sharp bounds for eigenvalues of the generalized k, m-step Fibonacci matrices

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Abstract

For two given integers $k = 1, 2, \ldots, m = 0, 1, \ldots$, and the nonnegative real constants $c_1, c_2, \ldots, c_k$, the n-th term, f_n of the *generalized k, m-step Fibonacci sequence* is given by the recursive formula

$$\begin{aligned} f_n &= c_1 f_{n-m-1} + c_2 f_{n-m-2} + \cdots + c_k f_{n-m-k} \\ &= \sum_{j=m+1}^{k+m} c_{j-m} f_{n-j}, \quad \text{for every } \; n \geq k+m+1, \end{aligned} \tag{1}$$

with

$$f_1 = f_2 = \ldots = f_{k+m} = 1.$$

Using (1) the generalized k, m-step Fibonacci sequence can be represented by the *generalized k, m-Fibonacci matrices*, which are defined in [1] and some bounds for the spectral radius of the matrices are discussed. In this paper, the powers of the generalized k, m-step Fibonacci matrices are investigated and closed formulas for their entries are derived, related to the suitable terms of the k, m-step Fibonacci sequences in order to develop the properties of the irreducibility and primitivity of the Fibonacci matrices. New upper and lower bounds for the spectral radius and the modulus of the remaining eigenvalues of the generalized k, m-Fibonacci matrices are presented. Some known results for the nonnegative matrices in the literature are generalized and improved, [2, 3, 4, 5]. Applications of the results are given in the tridiagonal k-Toeplitz matrices.

References

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