

The Computation of the Greatest Common Divisor of Three Polynomials

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Abstract

This paper considers the computation of the greatest common divisor GCD of three polynomials, $f(y)$, $g(y)$ and $h(y)$, using the Sylvester resultant matrix and its subresultant matrices. It is shown that there are two variations of the subresultant matrices for this problem, and that they have 2×3 and 3×3 partitioned structures. The order of the polynomials for the 2×3 partitioned structure is important because each ordering yields subresultant matrices of different dimensions. Furthermore, the magnitudes of the entries of the matrices may differ significantly, and their numerical ranks are not consistent. It is therefore necessary to consider the optimal sequence of the subresultant matrices, and this issue is addressed. Also, it is shown that $f(y)$, $g(y)$ and $h(y)$ must be preprocessed before computations are performed on their Sylvester matrix and its subresultant matrices. Computational examples are presented and the singular values of the 2×3 and 3×3 forms, and the variation of the singular values between the different partitioned structures of the 2×3 forms, are shown. It is also shown that it is important to include the preprocessing operations because it yields an approximate GCD and coprime polynomials with much smaller errors than the errors obtained when the preprocessing operations are omitted.
