On the positive definite solutions of the nonlinear matrix equations $X^p = A \pm M^T(X^{-1}\#B)^{-1}M$

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Abstract

The two nonlinear matrix equations $X^p = A+M^T(X^{-1}\#B)^{-1}M$ and $X^p = A\pm M^T(X^{-1}\#B)^{-1}M$ are studied, where $p \geq 1$ is a positive integer, M is an $n \times n$ nonsingular matrix, A is a positive semidefinite matrix and B is a positive definite matrix. We call $C\#D$ the geometric mean of positive definite matrices C and D. We show the existence and uniqueness of the nonlinear matrix equations. Estimates of the positive definite solution are given. Iteration method for finding the numerical solution is proposed.

References

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