Generalized SR calculus for derivatives of split-quaternionic functions

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Abstract

This paper is to give the relation between the derivatives of split-quaternion valued functions in $\mathbf{S}$ and the corresponding functions of four real variables in $\mathbb{R}^4$, in order to obtain a quaternion extension of the $\mathbf{HR}$ calculus termed the $\mathbf{SR}$ calculus. Due to non-commutativity of split-quaternion product, we induces the $\mathbf{SR}$ calculus as the left- and right-hand versions of derivatives of split-quaternionic functions. In particular, we show that for real functions of split-quaternion variables the left and right $\mathbf{SR}$ derivatives are identical. That is, the use of the left/right $\mathbf{SR}$ derivative does not affect to practical applications of (split-)quaternion optimization. Finally, we consider several fundamental theorems in (split-)quaternion calculus, based on the $\mathbf{SR}$ derivatives to enable expansion to (split-)quaternions in practical applications.

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