Coupled multipoint flux and multipoint stress mixed finite element methods for poroelasticity

Ivan Yotov[1]

[1] Department of Mathematics, University of Pittsburgh, Pittsburgh, PA 15260, USA; yotov@math.pitt.edu

Abstract

We discuss mixed finite element approximations for the Biot system of poroelasticity. We employ a weak symmetry elasticity formulation with three fields - stress, displacement, and rotation. We study two elasticity formulations, with poroelastic and elastic stress as primary unknown, respectively. Stability bounds and error estimates are derived for both formulations for arbitrary order mixed spaces. We further develop a method that can be reduced to a cell-centered scheme for the displacement and the pressure, using the multipoint flux mixed finite element method for flow and the recently developed multipoint stress mixed finite element method for elasticity. The methods utilize the Brezzi-Douglas-Marini spaces for velocity and stress and a trapezoidal-type quadrature rule for integrals involving velocity, stress, and rotation, which allows for local flux, stress, and rotation elimination. We perform stability and error analysis and present numerical experiments illustrating the convergence of the method and its performance for modeling flows in deformable reservoirs.