Distance to $\mathcal{R}$-Uncontrollability

George Miminis[1]

[1] Dept. of Computer Science, Memorial Univ. of Newfoundland, St. John's, NL, CANADA, A1B 3X7

Abstract

We consider the continuous time descriptor system $E\dot{x}(t) = Ax(t)+Bu(t)$, denoted by (E, A, B), with $E \in \mathbb{R}^{n\times n}$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $t \geq 0$ representing time. A fundamental concept associated with descriptor systems is that of controllability. Intuitively, this is the ability of x to move from an initial to a final value in finite time by some control action $u(t)$. If E is allowed to be singular, x is restricted to the so-called *reachable* subspace $\mathcal{R} \subseteq \mathbb{R}^n$. This gives rise to the concept of controllability within $\mathcal{R}$, termed $\mathcal{R}$-controllability, and to the concept of the *distance* of (E, A, B) from the nearest $\mathcal{R}$-uncontrollable system, also termed the *radius* of (E, A, B).

In this work we concentrate on a special kind of a descriptor system termed the *semiexplicit* system (J, A, B), where $J = diag(I, 0)$. Often, descriptor systems appear naturally in this form, but also any system (E, A, B) is equivalent to some semiexplicit system via an equivalence relation $(J, A, B) \equiv diag\left(\Sigma^{-1}, I\right) Q^T (E, A, B)\, diag(P, P, I)$, using for example the singular value decomposition $Q^T EP = diag(\Sigma, 0)$ of E. The *radius* μ of (J, A, B), can be defined, and be shown equal to:

$$\begin{aligned} \mu &\equiv \min_{(\delta A, \delta B)\in\mathbb{C}^{n\times(n+m)}} \|(\delta A, \delta B)\|_2 : (J, A+\delta A, B+\delta B) \text{ is } \mathcal{R}\text{-uncontrollable} \\ &= \min_{\lambda\in\mathbb{C}} \underbrace{\sigma_{\min}(B, A-\lambda J)}_{\sigma(\lambda)}, \text{ where } \sigma_{\min} \text{ denotes the smallest singular value.} \end{aligned}$$

The contribution of this research is the efficient estimation of μ without resorting to algorithms that estimate the radius of a general descriptor system since they all consider, as they should, perturbations on E and consequently give rise to algorithms that are unnecessarily complicated for semiexplicit systems. Furthermore some of them do not allow E to be singular. On the other hand, with the exception of one paper, which will be discussed, none of the papers that estimate the radius of (I, A, B) can be altered, at least in a straightforward way, in order to handle (J, A, B), where J is allowed to be singular.

The study of $\sigma(\lambda)$ shows that it possesses several local minima clustered in a specified area. To address this, we developed a novel algorithm for the computation of the global minimum of $\sigma(\lambda)$, which we termed the *feedback search algorithm*. We have implemented this algorithm in MATLAB and applied it successfully on $\sigma(\lambda)$ and on two other related functions using several random examples as well as examples from the literature.