Computation of the matrix logarithm using the double exponential formula

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Abstract

We consider computation of the matrix logarithm based on the integral representation

$$\log(A) = \int_0^1 (A - I)[t(A - I) + I]^{-1}\, \mathrm{d}t \tag{1}$$

where A is a square matrix whose eigenvalues do not lie on the closed negative real axis. The matrix logarithm arises in many applications such as von Neumann entropy and image registration.

The efficiency of computing the matrix logarithm of (1) by numerical integration depends on the choice of quadrature formula. The Gauss–Legendre quadrature is considered as one choice for (1) (e.g. [1]), because the Gauss–Legendre quadrature for (1) coincides with Padé approximation for $\log(A)$ (see, e.g. [2, p. 274]). However, when the condition number of A is large, the Gauss–Legendre quadrature may not be the best choice.

In this talk, we consider the Double Exponential (DE) formula instead of Gauss–Legendre quadrature. As compared with the Gauss–Legendre quadrature, the DE formula usually works well even if integrands have endpoint singularities, and the DE formula improves accuracy at low cost (without recalculating abscissas-weight pairs). On the other hand, since the DE formula changes the integral interval into $(-\infty, \infty)$, we need to estimate the truncation error and give a promising finite interval for practical use. We propose an algorithm including the truncation error estimation, and show some numerical results to confirm its efficiency.

References

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