Finite difference Eulerian-Lagrangian schemes for hyperbolic problems with discontinuous flux and stiff source

Eduardo Abreu[1], Panters Bermudez[2], Vitor Matos[3], John Perez[4]

[1] *University of Campinas, Department of Applied Mathematics, Brazil*
[2] *Fluminense Federal University, Brazil*
[3] *University of Porto, Portugal*
[4] *Metropolitan Institute of Technology, Colombia*

Abstract

We formally develop a family of finite-difference shock-capturing schemes. This work also considers the questions of convergence of finite-difference approximations towards the entropic weak solution (correct shocks) of scalar, one-dimensional conservation laws with strictly convex and nonconvex flux functions. The finite-difference scheme is extended towards the viscosity solution of scalar, nonlinear multi-dimensional nonconvex (e.g., Buckley-Leverett) and convex (e.g., inviscid Burgers) model problems, which are presented and discussed. A new feature of the proposed method is the tracing forward to deal with balance laws and hyperbolic problems instead of trace backward in time over each *time step interval*. Indeed, we do not use approximate/exact Riemann solvers, and we do not use upwind source term discretizations either. Thus, we have a simple and fast Lagrangian-Eulerian solver for hyperbolic problems with discontinuous flux and stiff source. Our approach is based on a space-time Eulerian-Lagrangian framework introduced in [1]. Numerical tests show the robustness and accuracy of the method for a wide range of non-trivial applications available in the literature [2, 3, 4].

References

[1] E. Abreu, W. Lambert, J. Perez and A. Santo, A new finite volume approach for transport models and related applications with balancing source terms, Mathematics and Computers in Simulation 137 (2017) 2-28.

[2] F. Coquel, J-M Hérard, K. Saleh and N. Seguin, A robust entropy-satisfying finite volume scheme for the isentropic Baer-Nunziato model, ESAIM: Mathematical Modelling and Numerical Analysis 48(1) (2014) 165-206.

[3] L. Gosse, Computing Qualitatively Correct Approximations of Balance Laws Exponential-Fit, Well-Balanced and Asymptotic-Preserving, SIMAI Springer (2013).

[4] J. M. Greenberg, A. Y. LeRoux, R. Baraille, and A. Noussair, Analysis and Approximation of Conservation Laws with Source Terms, SIAM J. Numer. Anal. 34(5) (1997) 1980-2007.