Approximate Greatest Common Divisor through factorization of matrices of special structure

M. Mitrouli[1], D. Triantafyllou[2]

[1] Department of Mathematics, University of Athens, Panepistemiopolis 15784, Athens, Greece, email: mmitoul@uoa.gr
[2] Department of Mathematics and Engineering Sciences, Hellenic Military Academy, 16673, Vari, Greece, email: dtriant@sse.gr

Abstract

The Greatest Common Divisor (GCD) of polynomials is needed in many real applications such as Image Processing, Secret Sharing Schemes, Networks, Control Theory and has attracted the interest of researchers for many years. The computation of the degree and the coefficients of the GCD of polynomials is a hard task. Noise and measurement errors in initial data can lead to wrong results. Thus, it is of interest the relaxation of the notion of the exact GCD and the computation of an approximate GCD (AGCD) of polynomials. In this paper, we present methods for the computation of the degree and the coefficients of the AGCD of several polynomials. The approaches of this work are based on matrices of special structure such as Generalized Sylvester and Bezoutians. Classical methods such as LU, QR and RRQR facrorization are used and appropriately modified in order to be applied to the special form of the matrices reducing the required computational complexity of the procedures. The error analysis of the methods guarantees the stability of the presented algorithms. Many numerical experiments testing and comparing the methods conclude to useful remarks.

References

[1] T. Beelen and Van Dooren, P., A pencil approach for embedding a polynomial matrix into a unimodular matrix, SIAM J. Matrix Anal. Appl., 9, 1988, pp. 77–89.

[2] D. A. Bini and P. Boito, A Fast Algorithm for Approximate Polynomial GCD Based on Structured Matrix Computations, Operator Theory: Advances and Applications, 199, 2010, pp. 155–173.

[3] R. M. Corless and S. M. Watt and L. Zhi, QR Factoring to compute the GCD of Univariate Approximate Polynomials, IEEE Transactions on Signal Processing, 52, No. 12, 2004, pp. 3394–3402.

[4] G. M. Diaz-Toca and L. Gonzalez-Vega, Barnett's theorems about the greatest common divisor of several univariate polynomials through Bezout-like matrices, J. Symbolic Computation, 34, 2002, pp.59–81.

[5] N. Karcanias, Invariance properties and characterisation of the greatest common divisor of a set of polynomials, Int. J. Control, 46, 1987, pp. 1751–1760.

[6] N. Karcanias and M. Mitrouli and D. Triantafyllou, Matrix Pencil methodologies for computing the greatest common divisor of polynomials: Hybrid algorithms and their performance, Int. Journ. of Control, 79, No. 11, 2006, pp. 1447–1461.

[7] I. S. Pace and S. Barnett, Comparison of algorithms for calculation of g.c.d of polynomials, Int. J. Control, 4, No. 2, 1973, pp. 211–216.

[8] T. Sasaki and M. Sasaki, Polynomial remainder sequence and approximate GCD, ACM SIGSAM Bull., 31, 1997, pp. 4–10.

[9] J. R. Winkler and X. Lao, The calculation of the degree of an approximate greatest common divisor of two polynomials, J. Comp. Appl. Math., 235, 2011, pp. 1587–1603.