Is Lanczos tridiagonalization essential for solving large eigenvalue problems?

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Abstract

In this talk we present a new type of restarted Krylov methods for calculating peripheral eigenvalues of a large sparse symmetric matrix, G. The new framework avoids the Lanczos tridiagonalization algorithm and the use of polynomial filtering. This simplifies the restarting mechanism and allows the introduction of several modifications. Convergence is assured by a monotonicity property that pushes the computed Ritz values toward their limits.

One improvement lies in the rule for determining the starting vector of the Krylov matrix that is built at each iteration. This rule is much simpler and intuitive than rules that are based on polynomial filtering. The Krylov matrix is generated by a three term recurrence relation that is based on MGS orthogonalization. These modifications lead to fast rate of convergence.

The differences between the new approach and the Lanczos approach become significant in inner-outer iterative methods which are using inexact inversions to compute small eigenvalues. The idea is to replace G with an approximate inverse of G, which is obtained by solving a linear system. So each column of the Krylov matrix requires the solution of a new linear system whose matrix is G. In this case Lanczos methods compute Ritz pairs of the approximate inverse of G, while our method computes Ritz pairs of G. In inner-outer methods most of the computation time is often spent on the inner iterations. This feature increases the appeal of the new approach, as the extra time that it pays for deserting the Lanczos algorithm becomes negligible.

The new approach is easily adapted to compute a partial SVD of a large sparse m x n matrix, A . Here it avoids the popular Lanczos bidiagonalization process. Instead the Krylov matrix that is built at each iteration is generated by using the cross-product matrix, ATA , or an approximate inverse of this matrix. Yet the singular triplets are computed from Ritz triplets of A. When using the cross-product matrix to compute low-rank approximations there is no need to update estimates of right singular vectors, which leads to considerable saving of storage.

The experiments that we have done are quite encouraging. In many cases the algorithm achieves the required accuracy within a remarkable small number of iterations.