Acceleration of iterative regularization methods by delta-convex functionals in Banach spaces

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Abstract

We consider a linear functional equation $Ax = y$ where $A : X \longrightarrow Y$ is an ill-posed operator between two complete normed linear functional spaces X and Y. The variational approach involves the minimization of a Tikhonov-type regularization functional $\Phi_\alpha : X \longrightarrow \mathbb{R}$ defined as

$$\Phi_\alpha(x) = \frac{1}{p}\|Ax - y\|_Y^p + \alpha\mathcal{R}(x)\,,$$

where $p > 1$, $\mathcal{R} : X \longrightarrow \mathbb{R}$ is a convex penalty term which quantifies the "non-regularity" of x, and $\alpha > 0$ is the regularization parameter which balances between data fitting and stability.

In the simplest case, referred as (basic) Tikhonov regularization, both X and Y are Hilbert spaces, $p = 2$ and $\mathcal{R}(x) = \frac{1}{2}\|x\|_X^2$. In the last two decades, several extensions to Banach space settings have been proposed in the literature to reduce the over-smoothness effects of the basic Hilbertian approach, also aimed at improving the sparsity, or at enforcing non-negativity or other special constraints. We just mention some special weighted Lebesgue spaces L^p or Sobolev spaces $W^{k,p}$, with $1 < p < +\infty$ [4].

In this talk, we discuss a special Tikhonov-type functional Φ_α whose penalty term $\mathcal{R}$ is model-dependent, that is, $\mathcal{R}$ explicitly depends on the operator A which characterizes the functional equation [3]. In addition, the functional is no longer convex as in the conventional setting, but in our proposal is delta-convex; i.e., it is representable as a difference of two convex terms. We will show that the proposed delta-convex functional allows us to speed up the convergence of iterative gradient minimization algorithms. This acceleration technique, which we call as "irregularization", is useful for large scale equations arising in image restoration [1]. Then, an extension of the algorithm to the unconventional variable exponent Lebesgue space $L^{p(\cdot)}$ [2] is also analyzed and numerically tested, aimed at providing a pointwise and adaptive control of the level of regularization.

References

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