Regularization preconditioners for frame-based image deblurring

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Abstract

We are interested in fast and stable iterative regularization methods for image deblurring problems with space invariant blur. The associated coefficient matrix has a Block Toeplitz Toeplitz Blocks (BTTB) like structure depending on the boundary conditions imposed on the imaging model. In the literature, several strategies have been proposed in the attempt to define proper preconditioner for iterative regularization methods that involve such linear systems. Usually, the structure of the preconditioner is chosen Block Circulant with Circulant Blocks (BCCB) because it can be efficiently exploited by Fast Fourier Transform (FFT). Nevertheless, for ill-conditioned problems, it is well known that BCCB preconditioners cannot provide a strong clustering of the eigenvalues. Moreover, in order to get an effective preconditioner, it is crucial to preserve the structure of the coefficient matrix. On the other hand, thresholding iterative methods are recently successfully applied to image deblurring problems, exploiting the sparsity of the image in a proper wavelet domain.

Motivated by the results of recent papers [2, 3], we combine a nonstationary preconditioned iteration [1] with the modified linearized Bregman algorithm (MLBA) and proper regularization operators. Namely, the proposed algorithms are made of an outer step of a nonstationary Landweber iteration, preconditioned by a Tikhonov-type preconditioner, and an inner step of thresholding. The first one Tikhonov-type preconditioner is chosen as a matrix function of the corresponding BCCB approximation of the original square matrix operator, obtained imposing the proper boundary conditions that arise from the problem itself. The second one is chosen as a regularized version of the BCCB approximation but preserving the structure of the original square matrix operator. We prove that all the algorithms are regularizing and convergent, both in the free-noise case and in the noise case. Finally, several numerical experiments shows the consistency of our methods in terms of speed and quality of the restorations.

References

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