Solving nonlinear systems of PDEs with the Partition of Unity - RBF method via the trust-region algorithm.

Francisco Bernal[1], Elisabeth Larsson[2], Alfa R.H. Heryudono[3]

[1] *CMAP, École Polytechnique (Paris, France)*
[2] *Department of Information Technology, Uppsala University (Sweden)*
[3] *Department of Mathematics, University of Massachusetts Dartmouth (USA)*

Abstract

It is well known that globally-supported Radial Basis Function (RBF) collocation schemes have wonderful approximation properties of partial differential equations (PDEs), but are restricted to rather smallish problems. The recent Partition of Unity - RBF (PURBF) method [1] substantially lifts this limitation, while upholding the geometrical flexibility, straightforward coding, intuitive discretisation, and spectral accuracy of the global RBF approach. Moreover, when combined with the RBF-QR algorithm [2], the notorious trade-off between stability and accuracy also stops being a delicate issue. These features lend PURBF the potential of achieving the breakthrough of RBF methods into large-scale PDE applications.

In this work, we extend the trust-region scheme for nonlinear elliptic PDEs introduced in [3] to PURBF.

As a relevant application, we solve the steady flow of a viscous fluid past a cylinder, arguably the simplest—yet not trivial—example of fluid-structure interaction. As the Reynold's number grows, the eddy structure in the wake of the flow becomes increasingly involved. In order to adequately resolve it, the discretization must be refined in the recirculation area, while the computational domain must be extended further away from it.

We pose the relevant Navier-Stokes equations in natural variables and discretize them with the PURBF method. The resulting nonlinear system of collocation equations is optimally handled with the trust-region algorithm. The accuracy of the proposed numerical method is demonstrated by comparison with the reference solution [4] (up to $Re = 40$), and by monitoring the drag coefficient, beyond it.

References

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