Convergent Domain Decomposition Methods for Total Variation Minimization

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Abstract

Total variation regularisation is an important tool to solve inverse imaging problems. In particular, in the last decades, in the literature, there have been introduced many different approaches and algorithms for minimizing the total variation. These standard techniques are iterative-sequentially formulated and therefore not able to solve large scale simulations in acceptable computational time. For such large problems we need to address methods that allow us to reduce the problem to a finite sequence of subproblems of a more manageable size, perhaps computed by one of the standard techniques. With this aim, we introduce domain decomposition methods for total variation minimization. The main idea of domain decomposition is to split the space of the initial problem into several smaller subspaces. By restricting the function to be minimized to the subspaces, a sequence of local problems, which may be solved easier and faster than the original problem, is constituted. Then the solution of the initial problem is obtained via the solutions of the local subproblems by glueing them together. In the case of domain decomposition for the non-smooth and non-additive total variation the crucial difficulty is the correct treatment of the interfaces of the domain decomposition patches. Due to the non-smoothness and non-additivity, one encounters additional difficulties in showing convergence of more general subspace correction strategies to global minimizers. In particular there do exist counterexamples indicating failure of splitting techniques, see e.g. [1]. Nevertheless, in this talk we propose overlapping domain decomposition algorithms for the total variation minimization problem with the guarantee of convergence to a minimizer of the original functional [2]. The analysis is based on the relation between the primal (original) total variation minimization problem and its dual formulation. To the best of our knowledge, this is the first successful approach of a domain decomposition strategy for total variation minimization with a rigorous convergent analysis in an infinite dimensional setting. We provide numerical experiments, showing the successful application of the algorithms.

References

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