An $\ell^p - \ell^q$ minimization method with cross-validation for the restoration of impulse noise contaminated images

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Abstract

Discrete ill-posed problems arise in many areas of science and engineering. Their solutions, if they exist, are very sensitive to perturbations in the data. Regularization aims to reduce this sensitivity. Many regularization methods replace the original problem with a minimization problem with a fidelity term and a regularization term. Recently, the use of a p-norm to measure the fidelity term and a q-norm to measure the regularization term has received considerable attention, see, e.g., [2, 3, 4] and references therein. The relative importance of these terms is determined by a regularization parameter. When the perturbation in the available data is made up of impulse noise and a sparse solution is desired, it is often beneficial to let $0 < p, q < 1$. Then the p- and q-norms are not norms, thus the minimized functional is non-convex. For the minimization of such non-convex functional we resort to the algorithm proposed in [4].

The choice of a suitable regularization parameter is crucial for the quality of the computed solution. It is therefore important to develop methods for determining this parameter automatically, without user-interaction. In this talk we discuss two approaches based on cross validation for determining the regularization parameter in this situation. Computed examples that illustrate the performance of these approaches when applied to the restoration of impulse noise contaminated images are presented.

References

[1] A. Buccini and L. Reichel, An $\ell^p - \ell^q$ minimization method with cross-validation for the restoration of impulse noise contaminated images., Submitted, 2018.

[2] R. H. Chan and H. X. Liang, Half-quadratic algorithm for ℓ_p-ℓ_q problems with applications to TV-ℓ_1 image restoration and compressive sensing, in Proceedings of Efficient Algorithms for Global Optimization Methods in Computer Vision, Lecture Notes in Comput. Sci. # 8293, Springer, Berlin, 2014, pp. 78–103.

[3] C. Estatico, S. Gratton, F. Lenti, and D. Titley-Peloquin, A conjugate gradient like method for p-norm minimization in functional spaces, Numer. Math., 137 (2017), pp. 895–922.

[4] G. Huang, A. Lanza, S. Morigi, L. Reichel, and F. Sgallari, Majorization-minimization generalized Krylov subspace methods for ℓ_p-ℓ_q optimization applied to image restoration, BIT Numer. Math., 57 (2017), pp. 351–378.