Review and complements on dual mixed finite element methods for non-Newtonian fluid flow problems

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Abstract

For a steady and creeping flow of an incompressible quasi-Newtonian fluid, the most used formulation is based on the strain rate tensor. For Ω a bounded domain of $\mathbb{R}^2$ with a Lipschitz boundary Γ and a given mass forces $\boldsymbol{f}$ defined in Ω, the combination of the conservation equations leads to the Nonlinear Stokes problem:

$$\begin{cases} -\boldsymbol{div}\left(2\nu(|\boldsymbol{d}(\boldsymbol{u})|)\,\boldsymbol{d}(\boldsymbol{u})\right) + \nabla p = \boldsymbol{f} & \text{in } \Omega, \\ div\,\boldsymbol{u} = 0 & \text{in } \Omega, \end{cases} \tag{1}$$

where $\boldsymbol{u}$ and p, the unknowns of the problem, are the velocity and pressure, respectively. For $\nu_0 > 0$ a reference viscosity and r a fluid characteristic real parameter verifying $1 < r < \infty$, the viscosity function $\nu(\cdot)$, depending on $|\boldsymbol{d}(\boldsymbol{u})|$, is usually given by one of the two following famous models:

$$\forall x \in \mathbb{R}_+^*, \text{ Power law : } \nu(x) = \nu_0\, x^{r-2}, \text{ Carreau law : } \nu(x) = \nu_0 \left(1 + x\right)^{(r-2)/2}.$$

System (1) is supplemented by a set of boundary conditions.

The generalized Stokes problem (1) and its approximation by standard finite elements was widely studied. In these works, only the primal variables velocity and pressure are taken into account. But, for various reasons, one may need also information on the dual variables as velocity gradient $\nabla\boldsymbol{u}$, strain rate tensor $\boldsymbol{d}(\boldsymbol{u})$, extra-stress tensor $\boldsymbol{\sigma} = 2\nu(|\boldsymbol{d}(\boldsymbol{u})|)\,\boldsymbol{d}(\boldsymbol{u})$ etc. To do so, one need to build appropriate mixed formulations. The aim of this work is to present an exhaustive review on the available techniques using the mixed formulations for the problem (1), obtained for example in [2, 3, 4], and give some new results on the approximation of those problems.

References

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